COMPUTABLE REPRESENTATIONS OF PATTERNS IN ARCHITECTURAL SHAPES

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Abstract. This paper develops a schema theory based approach to the representation of patterns in architectural shapes. This representation is capable of computer implementation. The adequacy of any representation is critical for information processing in computer-aided design. Shape representation using shape elements and spatial relationships are elaborated and the construction of shape schemas and characteristics of shape schema are investigated. A representation for patterns in architectural shapes is demonstrated.

1. Introduction

The ability to learn concepts from given objects is important in design computing. It creates the possibility that the machine can be used as an aid to design artifacts using the learned concepts. In addition, based on these learned concepts, inferential interpretation and production can be possible. To support machine learning, the adequacy of any representation is critical. The representation language should describe the particular concept to support reasoning and generalisation appropriately (Ringland and Duce, 1988). Furthermore, it should have the ability to allow for the representation of what commences as implicit knowledge. It should have a well-organised form so that it is able to perform inferences correctly with predefined knowledge. From the result of inductive learning, new objects or events could be predicted or generated. Schema theory is employed to describe shapes and spatial relationships or patterns over shapes in architectural drawings. Schema representations of shapes have advantages in architectural design computation. They can abstract the information of shape without losing necessary information (Pinker, 1985). The information in schema descriptions is invariant in different positions. This paper mainly focuses on

the representation suitable for recognition of shape elements, their relationships and patterns.

2. Shape Representation

Shape representations and their manipulation have been studied in many areas of design computation, such as, shape grammars, morphology in spaces and logic representation (Stiny, 1975; Steadman, 1983; Coyne, 1988; Mitchell, 1990). Most of those researches are based on the notion that shapes are composed of subshapes and their spatial relationships. The subshape is a smaller shape that is embedded into the shape. A spatial relationship is the relationship between shapes in Cartesian space. This section is primarily concerned with representations of spatial relationships rather than physical shapes using schema theory.

2.1. SPATIAL RELATIONSHIP

A shape is composed of subshapes, and shapes may have relationships between each other. Shapes are recognised explicitly and implicitly. A shape that is initially represented explicitly is a *primary shape* and a shape that exists only implicitly in a primary shape is an *emergent shape* (Gero and Yan, 1994; Gero and Jun, 1997). Among those shapes, *bounded polyline shapes*[1] are considered as shapes in this paper. Relationships are spatial relationships between shapes. Isometric transformation relationships are the most fundamental spatial relationships upon which all shape representations, such as, topology, shape semantics and patterns, can be founded.

2.1.1. Shape Representation Convention
In groups of shapes, each shape can be described with respect to one another shape using spatial relationships, especially isometric transformation and topological relationships. The initial or primary shape can be represented as a referent, and a relationship is represented as a predicate in a propositional shape description with arguments.

$$S = R \{E, A \}$$

S : Shape
A : Arguments for relationship
E : Referent shape
R : Relationship between shapes

2.1.2. Spatial Relationships and Representations

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[1] A bounded polyline shape is an enclosed polyline shape, for any point on the boundary of which there exists at least one circuit composed of line segments which start from and end at the point without covering any line segment more than once.

Isometric transformation relationships: Isometric transformations are closed transformations (Mitchell, 1990) that transform one shape into another shape without losing any properties. These are relationships between congruent shapes. There are four kinds of isometric transformations: translation, reflection, rotation and scaling. Their representations are presented in Figure 1.
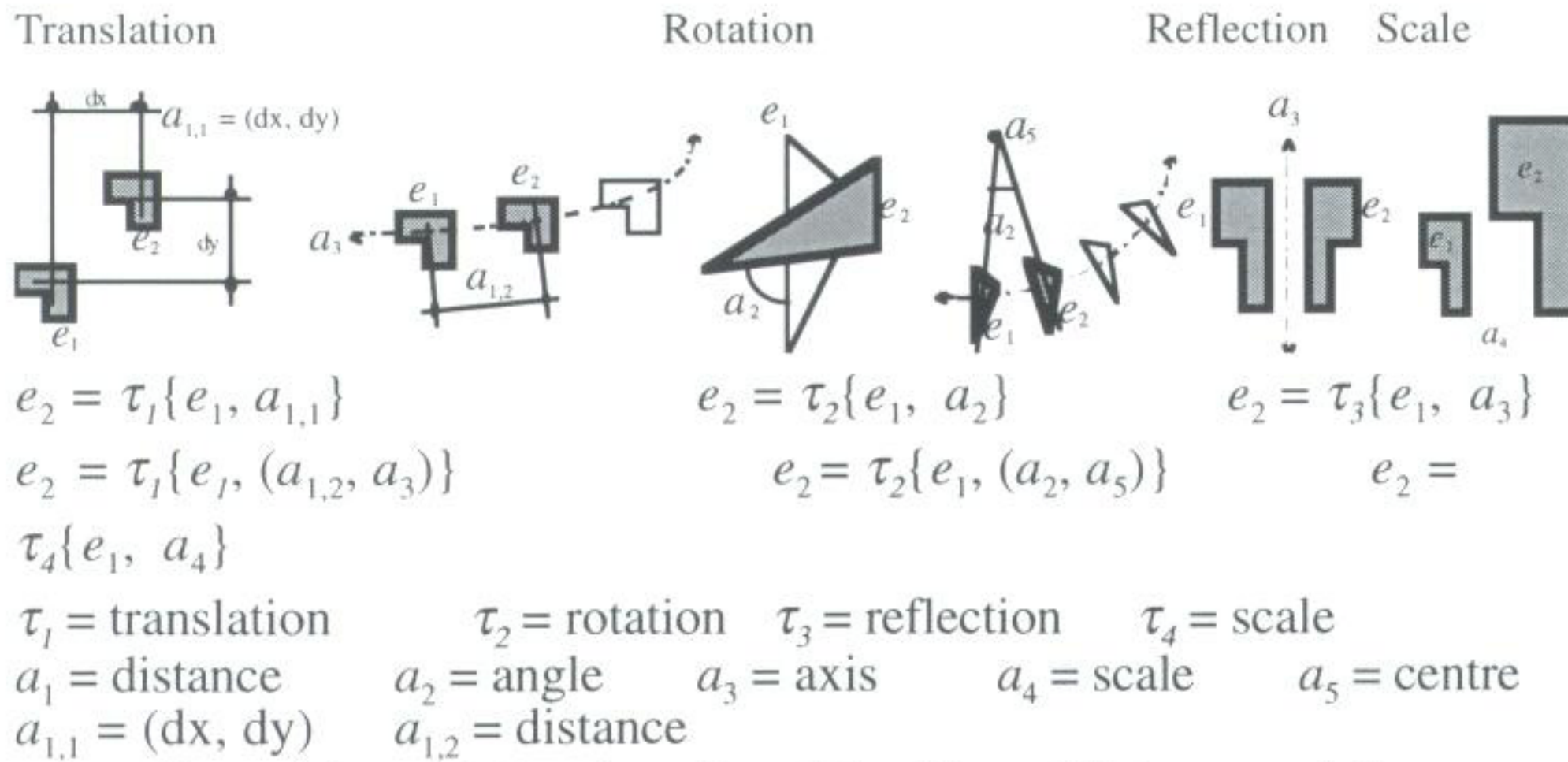
Translation	Rotation	Reflection Scale

$$e_2 = \tau_1\{e_1, a_{1,1}\}$$
$$e_2 = \tau_1\{e_1, (a_{1,2}, a_3)\}$$

$$e_2 = \tau_2\{e_1, a_2\}$$
$$e_2 = \tau_2\{e_1, (a_2, a_5)\}$$

$$e_2 = \tau_3\{e_1, a_3\}$$
$$e_2 = \tau_4\{e_1, a_4\}$$

τ_1 = translation $\quad$ τ_2 = rotation $\quad$ τ_3 = reflection $\quad$ τ_4 = scale
a_1 = distance $\quad$ a_2 = angle $\quad$ a_3 = axis $\quad$ a_4 = scale $\quad$ a_5 = centre
$a_{1,1}$ = (dx, dy) $\quad$ $a_{1,2}$ = distance

Figure 1. Isometric transformation relationships and their representations.

Topological relationships: Relationships between shapes are represented in terms of propositional relationships as well as isometric transformation relationships based on Euclidean space. The topological relationships specify

the relative positions between shapes, such as, front, back, top, bottom, left, right, and so on. For example, "A triangle is left of the square." and " A triangle is in front of the square." Where "top" and "front" are the topological predicates, "square" is a referent. Some relationships are variously interpreted by different propositions such as observers, objects and environments, and topological relationships are derived from basic shape relationship descriptions, such as, lower topological relationships and mathematical descriptions of spatial relationships (Olson and Bialystok, 1983). Topological relationships are represented as shown in Figure 2.
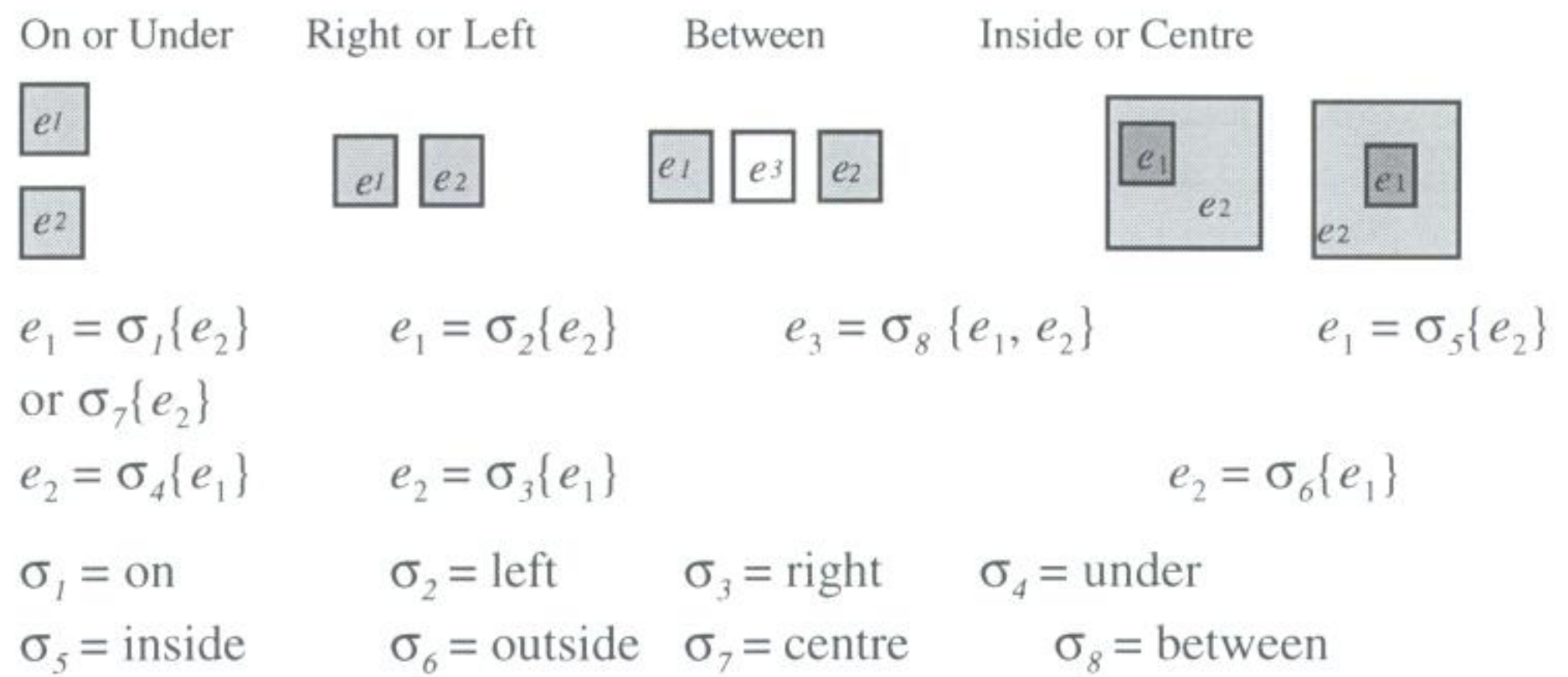
On or Under   Right or Left   Between   Inside or Centre

$$e_1 = \sigma_1\{e_2\}$$
or $\sigma_7\{e_2\}$

$$e_1 = \sigma_2\{e_2\}$$

$$e_3 = \sigma_8\{e_1, e_2\}$$

$$e_1 = \sigma_5\{e_2\}$$

$$e_2 = \sigma_4\{e_1\}$$

$$e_2 = \sigma_3\{e_1\}$$

$$e_2 = \sigma_6\{e_1\}$$

σ_1 = on σ_2 = left σ_3 = right σ_4 = under

σ_5 = inside σ_6 = outside σ_7 = centre σ_8 = between

Figure 2. Topological relationships and their representations.

2.2. SHAPE PATTERN REPRESENTATION

A pattern is a design in which a certain shape is repeated many times (Rowland, 1964). In this paper, similar shapes and spatial relationships, which are recursively arranged, are considered as shape patterns. Some shape patterns can be easily recognised and have names, such as, symmetry, rhythm, gradation, linear, central, etc., as exemplified in Figure 3.


Linear   Rotation Symmetry   Central   Gradation
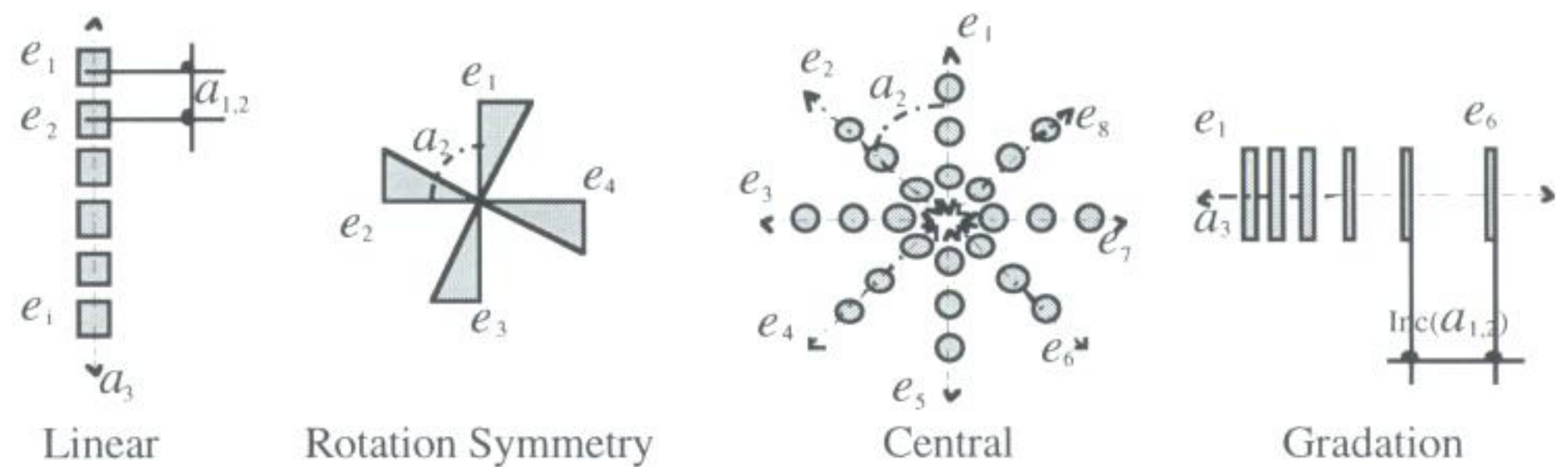
Linear: $\prod_{n=1}^{6} \tau_1\{e_i, (a_{1,2}, a_3)\}$ Central: $\prod_{n=1}^{8} \tau_2\{e_i, a_2\}$

Rotation Symmetry: $\prod_{n=1}^{4} \tau_2\{e_i, a_2\}$ Gradation: $\prod_{n=1}^{6} \tau_1\{e_i, \ [\mathrm{Inc}(a_{1,2}),$ $a_3]\}$

Figure 3. Shape patterns and their representations.

These patterns are represented using shape representation convention based on spatial relationships. For example, "linear" in Figure 3 is composed of 6 squares, every square (e_i) except the first one (e_1) has a translation relationship with the previous square (e_{i-1}), so the general representation of relationship between two shapes is $e_i = \tau_1\{e_{i-1}, (a_{1,2}, a_3)\}$. This group shape is

composed of six recursive translations from the initial shape (e_1). Recursive translations of elements starting from an initial shape to the nth shape can be represented using a *nesting operator* ($\prod_{n=1}^{i}$) that applies a transformation factor to elements recursively. Figure 4 shows more complex patterns and their representations.
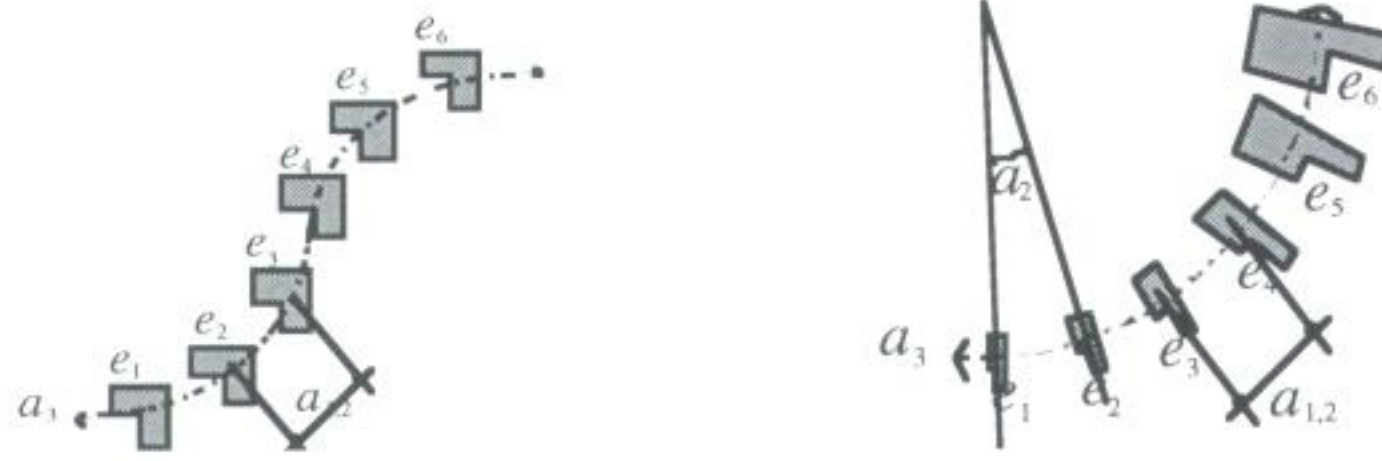
$$S = \prod_{i=1}^{n} \tau_1 \{e_i, (a_{1,2}, a_3)\} \qquad S = \prod_{i=1}^{n} \tau_4 \{\tau_1 [\tau_2(e_i, a_2), (a_{1,2}, a_3$$

$$)], a_4\}$$

Figure 4. Shape patterns and their representations.

2.3. RELATIONSHIPS OF RELATIONSHIPS

Relationships are composed from relationships as well as from form elements. Primitives or elements have their own internal relationships or semantics that can be identified with interpretations, such as, self-symmetry, centrality, regularity, linearity, etc. These relationships or semantics can be grouped together and specify a high level relationship recursively. Identified relationships are composed into a hierarchical tree structure that represents the shape.

Different shape groups may share the same relationships. In Figure 5, different sets of similar shapes are grouped together under the same linear transformation relationship. These relationships are congruent even though the elements are different.
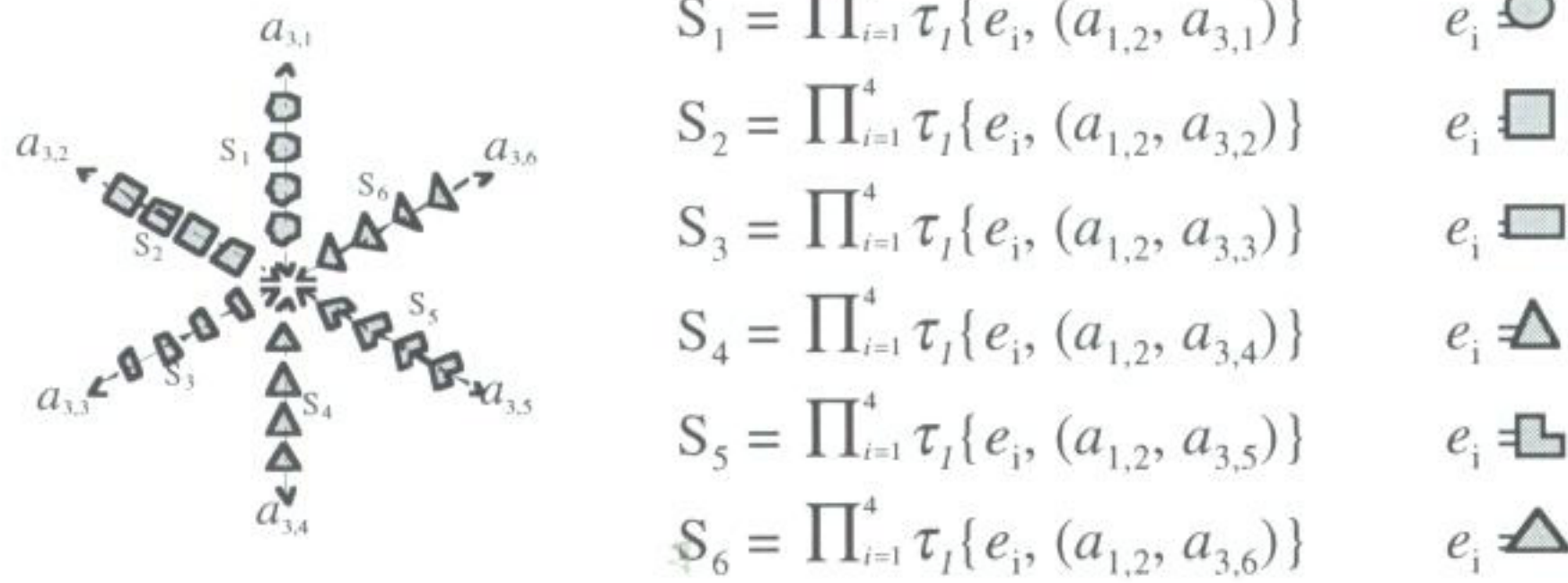

$$S_1 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,1})\} \qquad e_i \ \bigcirc$$

$$S_2 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,2})\} \qquad e_i \ \square$$

$$S_3 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,3})\} \qquad e_i \ \square$$

$$S_4 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,4})\} \qquad e_i \ \triangle$$

$$S_5 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,5})\} \qquad e_i \ \square$$

$$S_6 = \prod_{i=1}^{4} \tau_1 \{e_i, (a_{1,2}, a_{3,6})\} \qquad e_i \ \triangle$$

Figure 5. Six different sets of shapes are grouped together independently and specify the same translation relationship.

From six different shape groups, only relationships are considered to construct a higher level shape relationship. Properties of shape elements are disregarded, and shapes are considered as variables. Six congruent relationships in a pattern identify the 60° rotation relationship. So relationships are constructed not only from similar shapes but also from similar relationships, Figure 6.
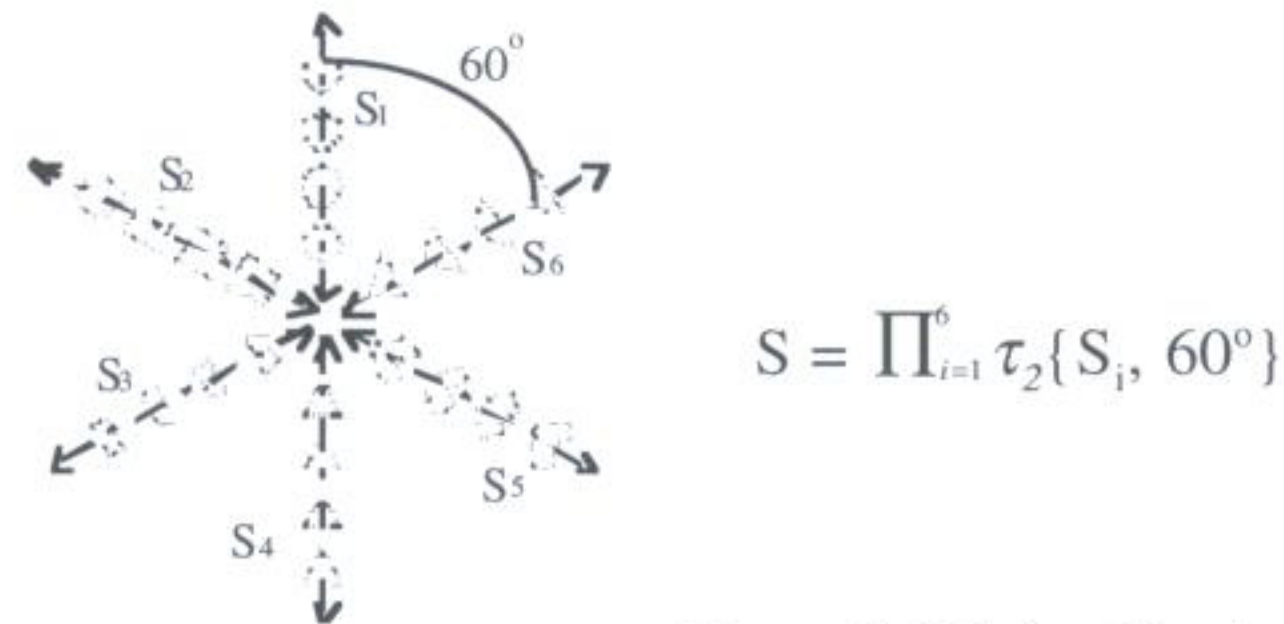

$$S = \prod_{i=1}^{6} \tau_2\{S_i, 60^\circ\}$$

Figure 6. Relationships from sets of shape groups are congruent, they are grouped together and identify a high level relationship.

3. Shape schema

Perceptual categories exist by virtue of similar structural descriptions (Palmer, 1977). In perceptual categories of shapes, objects are perceived not only by their properties, such as, colour, material, but also by their organising structures specified by spatial relationships. According to Rumelhart (1980), a schema is "a higher order relational structure for representing the generic concepts upon which all information processings depend". It is a network of inter-relationships that represents essential characteristics of things or concepts rather than a list of features. The network may be in a hierarchical tree structure with nodes and paths. It is generalised from multiple repetitions (Piaget, 1952) and describes a prototype (or a generic concept) for a group of things or situations (Minsky, 1975).

3.1. SCHEMA CONSTRUCTION

In this paper, a shape schema is an organised body of knowledge about spatial relationships between shapes that describes the patterns of shapes, syntactic structures of shapes and describes the characteristics of shapes. It is constructed from the repetition of similar elements, relationships and schemas, and is represented in a hierarchical tree structure. Identification of similarity between shape schemas supports comprehension of shapes and shape semantics and analogical reasoning about shapes.

In shape schema construction, a class of drawings is provided as input data, then primitives and their relationships are identified using predefined shape knowledge. Predefined shape knowledge is knowledge about pure shape, spatial relationships, shape emergence and semantics emergence and aesthetics properties. With these identified primitives and relationships, multiple conceptual shape descriptions for each shape are constructed in hierarchical tree structures. Multiple representations for a single shape are possible and shape descriptions from a class of shapes are generalised into shape schemas by inductive generalisation rules.

3.2. MULTIPLE REPRESENTATIONS

Schemas are constructed from repeated structural properties (or common structural properties) in a class of objects. A single object in a class can be represented in terms of different attributes, in addition, many different representations are possible by perceivers, by different composition methods from elements and by the object's internal complexity.

Primitive shapes are identified from the shape input data as well as emerged shapes. Primitive shapes are similar shapes to stored shapes in predefined knowledge, or peculiar shapes that are easily recognised. Properties of primitives are compared and similar shapes are grouped together and their spatial relationships are identified using inductive learning processes (Michalski, 1983). Various shape descriptions can be constructed from single shape objects in terms of different intentions and predefined shape knowledge. For example, two different sets of shapes are recognised in Figure 7, a set of small squares and three large squares with small rectangles. Two congruent squares in Figures 7(c) and (d) have reflection relationships about the horizontal or vertical axes as well as four congruent squares through a 90° rotation relationship, and three groups of four squares identify translation relationships. Also three large squares with small rectangles specify the same translation relationship in Figure 7(b).

Repeated shape schemas or shape descriptions extracted from multiple shape representations can be used for schema construction from a class of shape objects. In Figure 7, three representations from a single object share the translation relationship of three elements. It is a shape schema that is constructed from multiple representations.

(a)

(b)

(c)

(d)
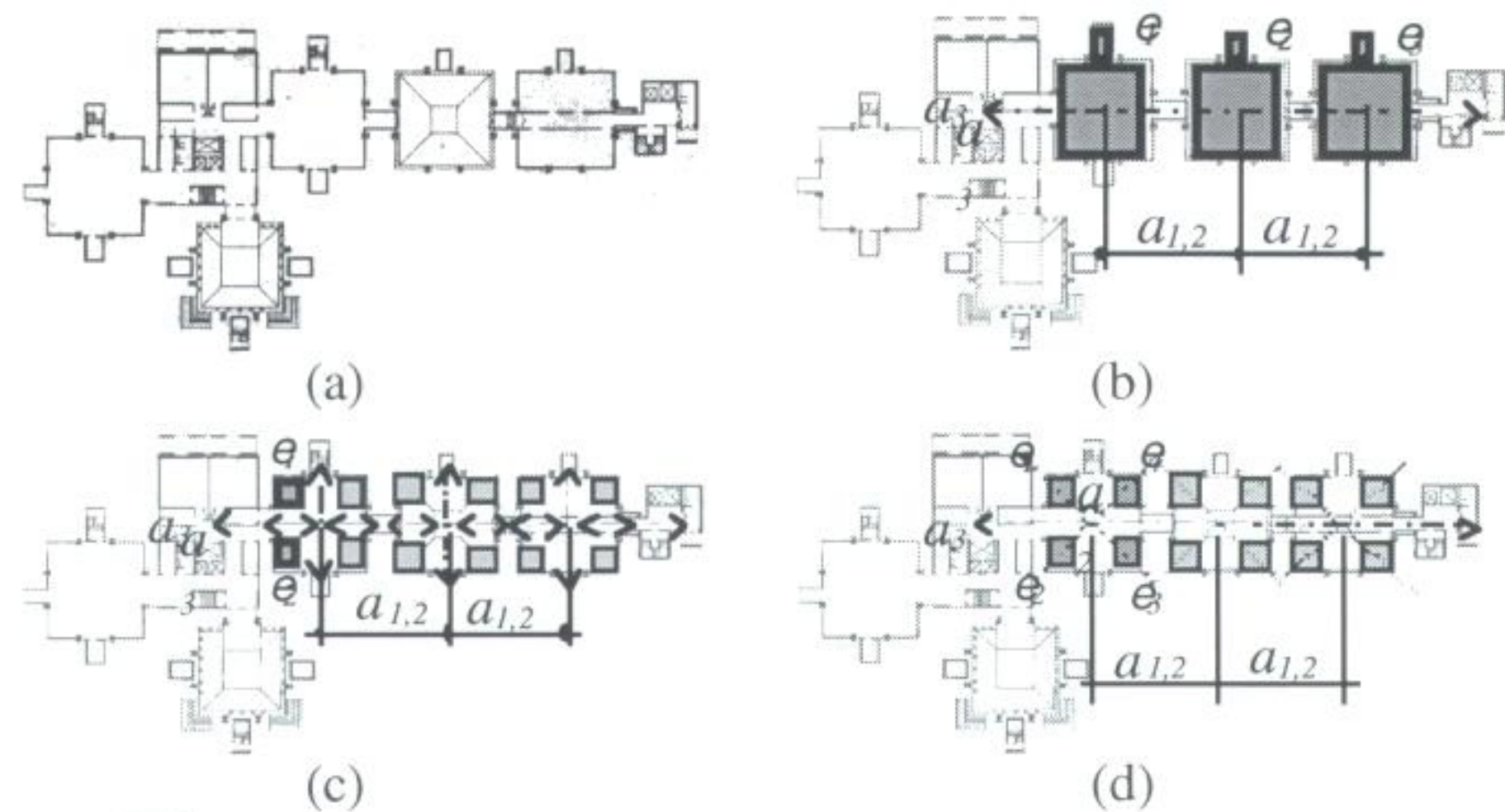
$$(b) = \prod_{i=1}^{3} \tau_1 \{ e_i, (a_{1,2}, a_3) \}$$

$$(c) = \prod_{k=1}^{3} \tau_1 \{ \prod_{j=1}^{2} \tau_3 [\prod_{i=1}^{2} \tau_3 (e_{i,j,k}, a_3), a_3], (a_{1,2}, a_3) \}$$

$$(d) = \prod_{j=1}^{3} \tau_1 \{ \prod_{i=1}^{4} \tau_2 (e_{i,j}, a_2), (a_{1,2}, a_3) \}$$

Figure 7. Multiple representations from a single shape: (a) Richard Research Institute by Louis I. Kahn, (b), (c) and (d) three different representations of the drawing in (a).

3.3. CHARACTERISTICS OF SCHEMA

Variability: A shape schema is not a representation of a specific shape, rather it represents spatial characteristics for a class of shapes. Each shape is a specific instantiation of the shape schema. According to Minsky (1975), the upper levels of the frame (schema) are fixed and represent unalterable truths about things, while lower levels consist of "terminals" or "slots" that are filled with specific instances or data. From the notion of schema as a network of elements and relationships, lower level schemas as well as lower level elements and relationships can be regarded as variables that can be instantiated. In shape schemas, shape elements and lower level relationship elements or schemas are considered as variables.

Embeddedness : One schema may be embedded in one or many others in various ways. A schema can be part of another schema, and many schemas can share a subschema, or a schema can be embedded within itself producing recursion.

4. Representation of patterns in architectural drawings

We can use the symbolic representations developed in the previous sections to describe, in a potentially computable form, complex architectural

drawings which are otherwise too difficult to represent as a simple conjunction of elements. Figure 8(a) shows an architectural plan of dormitories designed by Louis I. Kahn. Figures 8(b) and (c) show the shape elements and the symbolic representation of those elements and their relationships which go to make up this plan. Thus, the plan can be represented through a combination of translational and rotational transformations of constant elementary shapes.

(a)

(b) (c)
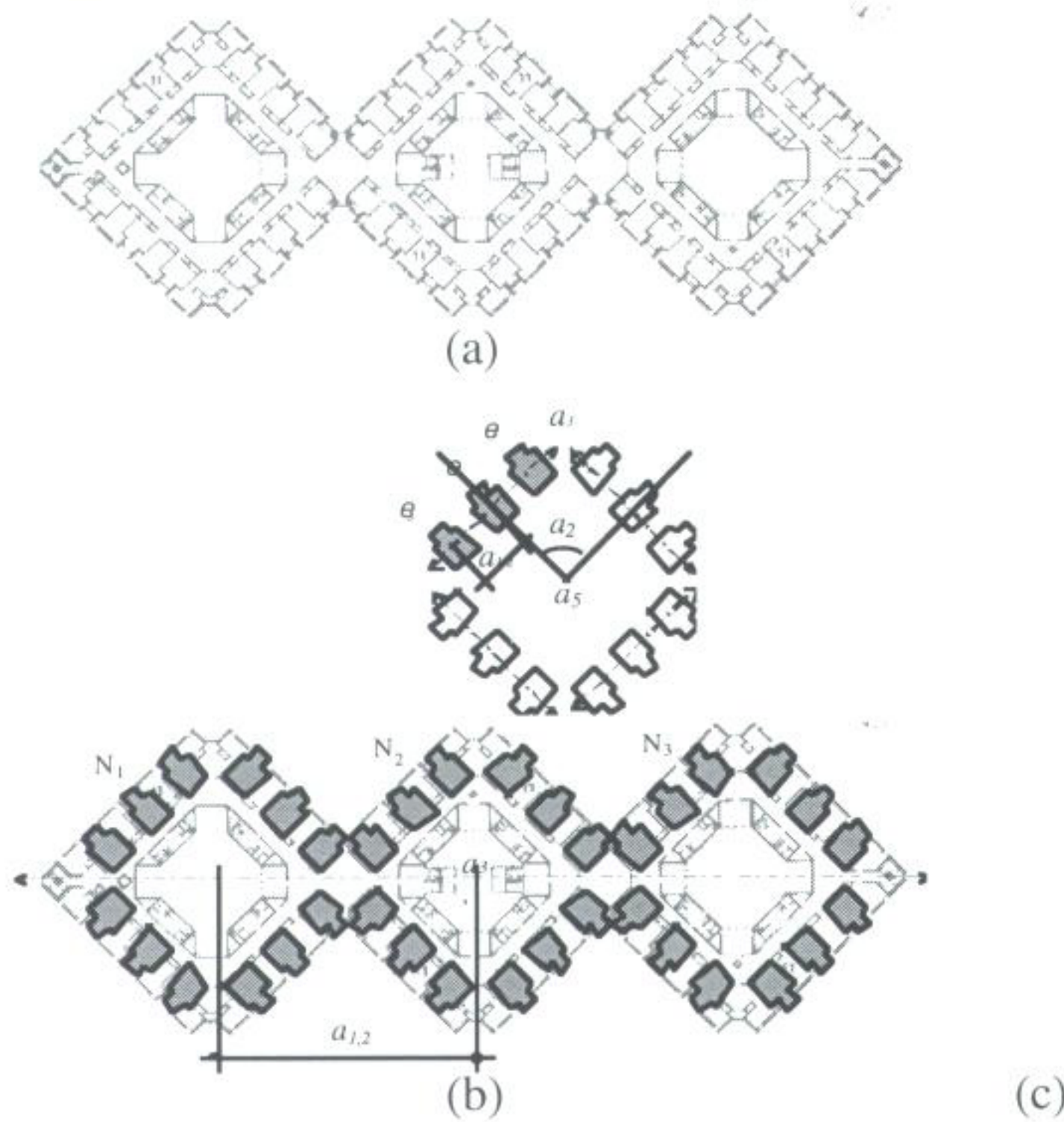
$$N = \prod_{j=1}^{4} \tau_2 (\prod_{i=1}^{3} \tau_1 \{ e_{i,j}, (a_{1,2}, a_3) \}, (a_2, a_5))$$

$$\text{Dormitories} = \prod_{k=1}^{3} \tau_1 \{ N_k, (a_{1,2}, a_3) \}$$

$$= \prod_{k=1}^{3} \tau_1 \{ \prod_{j=1}^{4} \tau_2 [\prod_{i=1}^{3} \tau_1 (e_{i,j,k}, (a_{1,2}, a_3)), (a_2, a_5)], (a_{1,2}, a_3) \}$$

Figure 8. Architectural drawing and representations: (a) dormitories by Louis I. Kahn, (b) and (c) representations of the drawing in (a).

Figure 9(a) shows a plaza plan of the Campidoglio designed by Michelangelo. Figures 9(b) and (c) show the shape elements and the symbolic representation of those elements and their relationships which go to make up this plaza plan. Thus, one representation of this plaza plan is through a combination of rotational and scaling transformations of the elementary shapes.
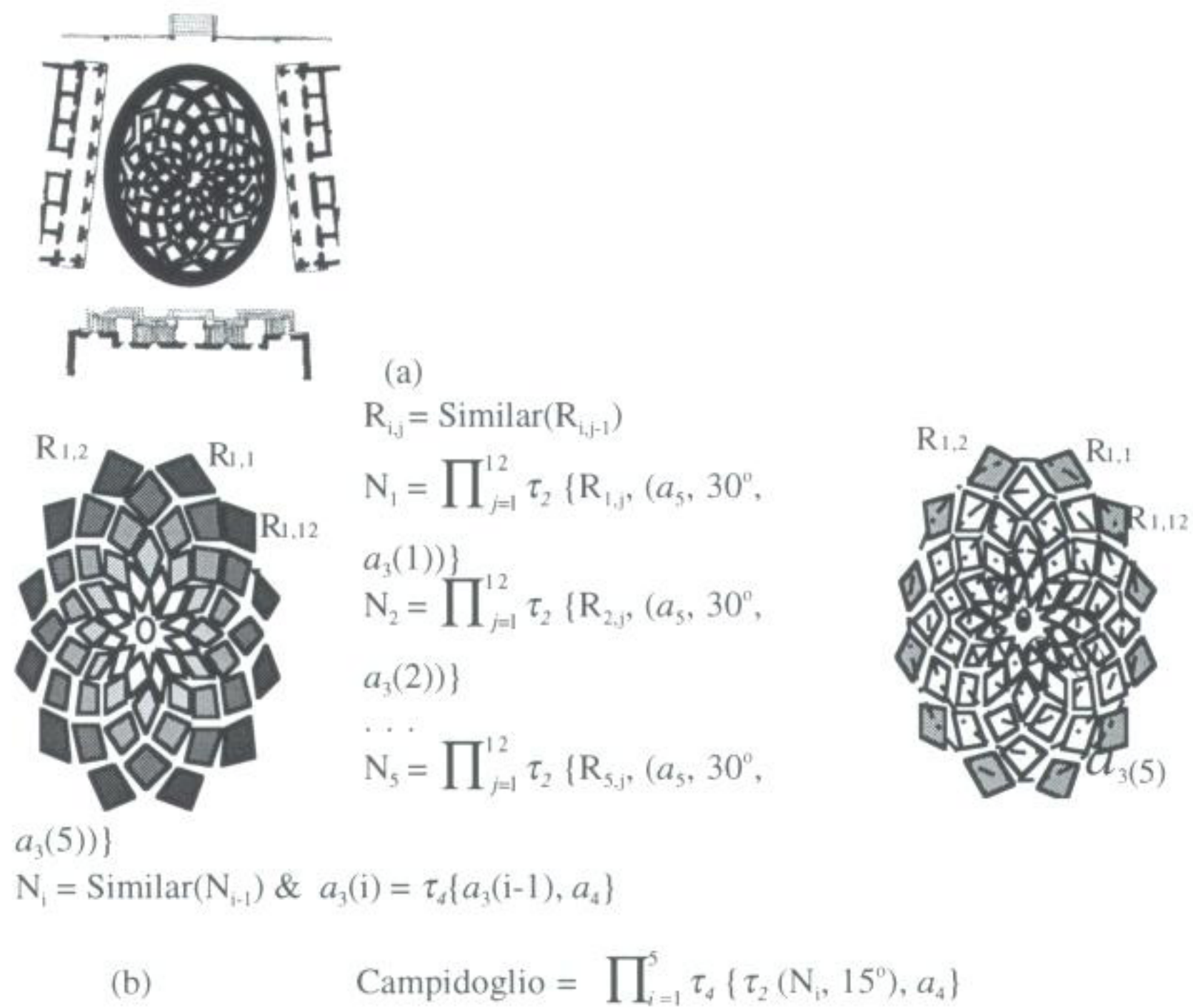
(a)

$R_{i,j} = \text{Similar}(R_{i,j-1})$

$N_1 = \prod_{j=1}^{12} \tau_2 \{R_{1,j}, (a_5, 30°,$

$a_3(1))\}$

$N_2 = \prod_{j=1}^{12} \tau_2 \{R_{2,j}, (a_5, 30°,$

$a_3(2))\}$

. . .

$N_5 = \prod_{j=1}^{12} \tau_2 \{R_{5,j}, (a_5, 30°,$

$a_3(5))\}$

$N_i = \text{Similar}(N_{i-1}) \ \& \ a_3(i) = \tau_4\{a_3(i-1), a_4\}$

(b) $\text{Campidoglio} = \prod_{i=1}^{5} \tau_4 \{\tau_2 (N_i, 15°), a_4\}$ (c)

Figure 9. Architectural drawings and representations: (a) The Campidoglio by Michelangelo, (b) and (c) representations of the drawing in (a).

5. Conclusion

In shape images, the concepts are carried by shape elements and their perceptual organisations, so called, shape schema. This paper has focused on the symbolic representation of shape relationships and schemas. Shape elements are primitive shapes or group of shapes. Their basic spatial relationships, such as, translation, reflection, rotation and scale, are represented with predicates and arguments from which visual patterns and perceptual design concepts are recognised. Low level elements, such as, primitive shapes, relationships and schemas, whose changes do not affect shape meanings, can be turned into variables. One schema may be embedded into one or many others in various ways.

Shapes with many attributes and relationships can be recognised and interpreted in various ways. However, this paper has focused on bounded polyline shapes and their spatial relationships, particularly, isometric transformation relationships and topology. Behaviours, functions and meanings of shapes have been excluded. Those properties need to be considered for the completion of shape representations.

The shape schema representation supports many areas in architectural computations, such as, analogical reasoning, style learning, categorisation

and study of style. According to Gentner (1989), the determination of similarity depends on relationships and attributes. Identification of similarity between shapes starts with comparing predicates and arguments from their representations. Shapes that share the same predicates and arguments can be considered as similar shapes. Style, as common features in a set of objects, can be learned by the computer using inductive generalisation theory. A class of shape descriptions is generalised in terms of inductive generalisation rules, which produce schemas that characterise the style. Conjunctive generalisation and disjunctive generalisation processes start from a set of shape descriptions and the results support categorisation. Schema knowledge and style knowledge about shapes can be invoked for the study of style.

Acknowledgments

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