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COMPLEXITY MEASURES OF DESIGN DRAWINGS AND THEIR APPLICATIONS

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Abstract

This paper presents an approach to the computational use of drawings through the development of complexity measures, defined as the information content of the description of a drawing's structure. As such it is the function of the representation used. The paper presents examples of the use of such measurements applied to design drawings of architects.

Introduction

The final outcome of the design process, irrespective what form that process takes, is usually a set of design drawings or a computer model of the resulting design. Apart from the resulting artifact itself the drawings or computer model are the only tangible resultants of the designer(s). These drawings or model are then used to construct, manufacture or fabricate the design. In the case where no artifact results, they are all that publicly remains of the design effort. Usually, such drawings are used indirectly during the design process by designers, particularly during conceptual design (Goldschmidt, 1994; Purcell and Gero, 1998: Suwa, Gero and Purcell, 1999). They are generally not used by an computational design support system. Given the importance of drawings to designers it seems surprising that there is so little use made of them by computational systems during or after the design process.

This paper presents an approach to the computational use of drawings. It does so through the development of complexity measures of design drawings. The complexity of a design drawing is defined as the information content of the description of its structure. As such it is the function of the representation used. These complexity measures are based on measuring the information content of a drawing by measuring its entropy. The complexity measures can then be used for both analysis and synthesis purposes.

Complexity Measures of Drawings

The complexity of a drawing can be measured by determining its information content. The higher the information content the higher its complexity. In formation content can be calculated from the entropy of the representation of the drawing. he entropic measures of a drawing require that the drawing be represented in a linear or graph theoretic form. This is carried using a qualitative representation of the drawing (Gero and Park, 1997). A drawing is converted into a set of landmarks. A landmark occurs whenever there is a change in some characteristic. The most obvious characteristic in a line-based drawing is a change in direction of a line. Such a change in direction of two adjacent line segments becomes a landmark and a qualitative measure used to represent that change. Other landmarks that are commonly used for drawings include change in relative length of two adjacent line segments and change in curvature of two adjacent line segments. These qualitative measures are used to convert the drawing into a symbol string representing aspects of those landmarks. The symbol string now forms the representation of the drawing and becomes the data for the measurement of complexity.

A number of measures of entropy and resultant complexity are possible depending on whether we are interested in individual drawings or sets of drawings. The simplest form is Shannon entropy (Shannon, 1948). Shannon entropy is a central notion in information theory. It is based on a simple statistical model of data, which assumes that they are generated by an ergodic Markov source, symbol after symbol (in the simplest case and feature after feature in the more general case). This generation is executed stochastically on the basis of the history of which symbols have been generated immediately before the current one. The entropy of a random variable q with the probability distribution $\mathrm{Prob}_M(q)$ is defined as

$$En = -\sum \mathrm{Prob}_M(q) \log \mathrm{Prob}_M(q). \qquad (1)$$

The model's estimate of the probability of generating q_i after q_j, $\mathrm{Prob}_M(q_i \mid q_j)$, is assumed to be constant. These conditional probabilities are estimated from a sample of the symbol string. The cross-entropy is then calculated as

$$En = -\sum_{i,j} P_S(q_i, q_j) \log \mathrm{Prob}_M(q_i, q_j) \qquad (2)$$

where $P_S(q_i, q_j)$ is the empirical probability of the symbol q_i following q_j in this sentence. Entropy is an ensemble-based measure that can only be calculated for an ensemble of similar (generated by the same source) sequence. This entropy forms the basis of the complexity of the drawing and is used to compare one drawing with another in a set of similarly produced drawings.

The comparison between two groups of designs within a design space can be carried out by constructing a Markov model for each group and then computing cross-entropy for each group with respect to another

$$En = -\sum_{i,j} P_T(q_i, q_j) \log \mathrm{Prob}_M(q_i, q_j) \qquad (3)$$

where $P_T(q_i, q_j)$ is the empirical probability of the symbol q_i following q_j in the other set (so the model comes from the first group and the empirical probabilities from the second group). In practice, when such analyses are carried out we normally use perplexity, *PP*, instead of cross-entropy. Perplexity (Pincus, 1991), *PP*, is defined as

$$PP = 2^{En}. \qquad (4)$$

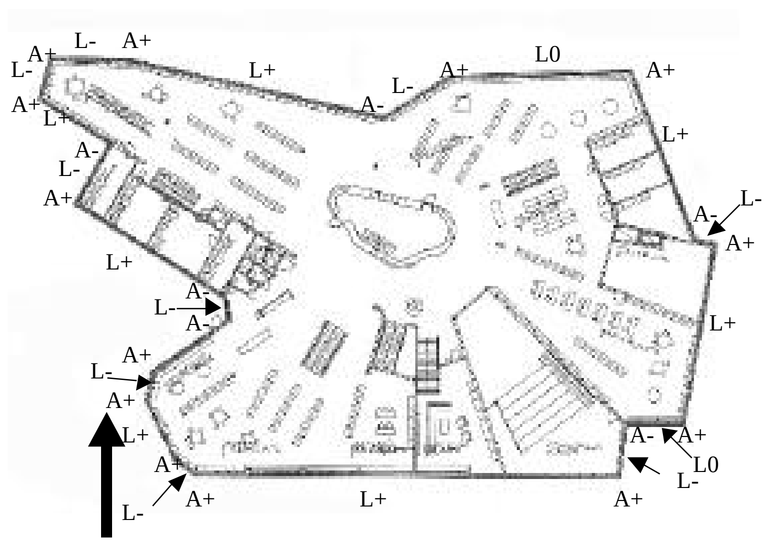
With these measures we are able not only to calculate the complexity of individual drawings as well as sets of drawings, we are able to use these measures to determine to make analytic statements about those drawings. For example, the higher the perplexity, the further apart are the two groups from each other and the less similar are the two groups of design drawings being compared. Thus, it becomes possible to compare sets of drawings from a single designer from different periods to determine if there has been a change in complexity. We are now in a position to compare the design drawings of two different designers to determine how similar or different they are.

Qualitative Representation of Drawings

We will use the published qualitative shape representation scheme called Q-codes (Gero and Park, 1997) as our canonical representation. Q-codes allow the outline of a shape to be encoded qualitatively in terms of the relative changes of directions at landmarks such as corners and the relative changes of lengths of adjacent sides. This converts the drawing into a circular string of Q-codes, Figure 1. This creates the common frame of reference for different drawings. The Q-code coding process follows formal guidelines developed in qualitative reasoning (de Kleer and Brown, 1984) and includes two stages:

- The set of landmarks is placed on the outline of a drawing. Landmarks are defined as points that are considered as distinguished by the coding procedure. Here we define them as points where the direction of the outline has a discontinuity on a coarse “macroscopic” scale. For example, there are 19 landmarks (“corners”) on the Aalto’s drawing that is shown in Figure 2.

- In a counterclockwise order each landmark is coded as {A+}, {A0} or {A-} correspondingly, depending on which of the intervals {(0,π), [0,0] or [π,2π)} the angle between two segments adjacent to this landmark belongs to; and each segment is coded as {L+} if its length is longer than the length of the previous segment, {L0} if they have the same length and {L-} if it is shorter than the previous one. The resulting symbol string is understood as a circular structure, that is, its last symbol is followed by the first one. An example of the Q-code coding is shown in Figure 1. Note that the granularity of the coding can be increased by reducing the sizes of the corresponding intervals.



{A+,L+,A+,L-,A+,L+,A+,L-,A-,L0,A+,L+,A+,L-,A-,L+,A+,L0,
A+,L-,A-,L+,A+,L-,A+,L-,A+L+,A-,L-,A+,L+,A-,L-,A-,L-,A+,L-}

Fig. 1: Q-code representation of a building plan by the architect Alvar Aalto. The angle in each corner is coded as {A+} if it is < π, as {A–} if it is > π and as {A0} if it is equal to π. The coding is carried out in a counterclockwise fashion, beginning with the corner to which the bold arrow is pointed. The length of each border segment is coded as {L+} if it is longer than the previous one, as {A-} if it is shorter and {A0} if they have the same lengths. The resulting string is understood as a circular structure, where the its two end points are connected.

These Q-codes are composed in a similar way to any natural language such as English. A sequence of Q-codes forms a word that may represent a shape pattern of significance such as a face of an object. A sentence, as a sequence of words, may describe an entire object.

Measuring the Drawings of Three Architects

We took a set of houses designed by three internationally famous twentieth century architects to determine both the complexity of the outlines of the plans of their house designs and then to use these designs as a basis for predicting each other's drawing complexity. The three architects were two American architects, Frank Lloyd Wright and Louis Kahn, and the Finnish architect Alvar Aalto.

The Markov model (of the 4-th order) was developed for each of these architect's selected house designs. The computations were carried out using the CMU-Cambridge statistical language modeling toolkit (Clarkson and Rosenfeld, 1997). Since it is well known that the quality of the model depends strongly on the size of the training sample we normalized the sample using the bootstrap re-sampling procedure (Efron and Tibshirami, 1993). Here a set of artificial samples was produced by re-sampling with replacement of the original sample. The results were then averaged over this set of samples. This forced all the training samples to have the same effective size. These models were evaluated (the perplexities were calculated) with respect to the original samples.

Table 1: The perplexities of Markov models developed for Aalto's and Kahn's architectural plans for houses and for Wright's prairie houses. The columns denote data sets and rows denote model developed using the outline shapes of plan drawings from corresponding architect as a training sample.

	Wright	Kahn	Aalto
Wright	1.78	23.6	20.9
Kahn	4.79	5.75	66.37
Aalto	7.52	33.22	5.90

The values along the diagonal refer to the model's perplexity for the data set in that row. From Table 1 one can see that the perplexities vary significantly, which confirms the discriminatory power of the approach. What these results indicate is that the outline plans of Wright's prairie houses are much less complex than the outline plans of the houses of either Kahn or Aalto (a perplexity value of 1.57 versus 5.75 and 5.90). Further, both Kahn's and Aalto's drawings are very similar in their complexity.

Along each row the values indicate the named architect's model of the complexity of their drawings as a predictor of the complexity of the architect's drawings. Thus, Wright's model is a poor predictor of Kahn and Aalto, particularly in relation to the model's ability to predict his own complexity. Kahn's model, somewhat surprisingly, is a good predictor of Wright and a very poor predictor of Aalto. Whilst Aalto's model is a reasonable predictor of Wright but a poor predictor of Kahn. The reason for such good predictions of Wright may be because the complexity of Wright's drawings is so low and, hence, the more complex models of Khan and Aalto have no difficulty in predicting it.

The qualitative representation we used in the examples includes an angle-based Q-code with three values {A-,A+,A0}. We calculated the relative value of the n-gram entropy for each of three bodies of house designs by dividing absolute entropy with the maximal entropy determined by the dimensionality of the alphabet of the representation. This is plotted in Figure 2, where n is the length of the code being used.

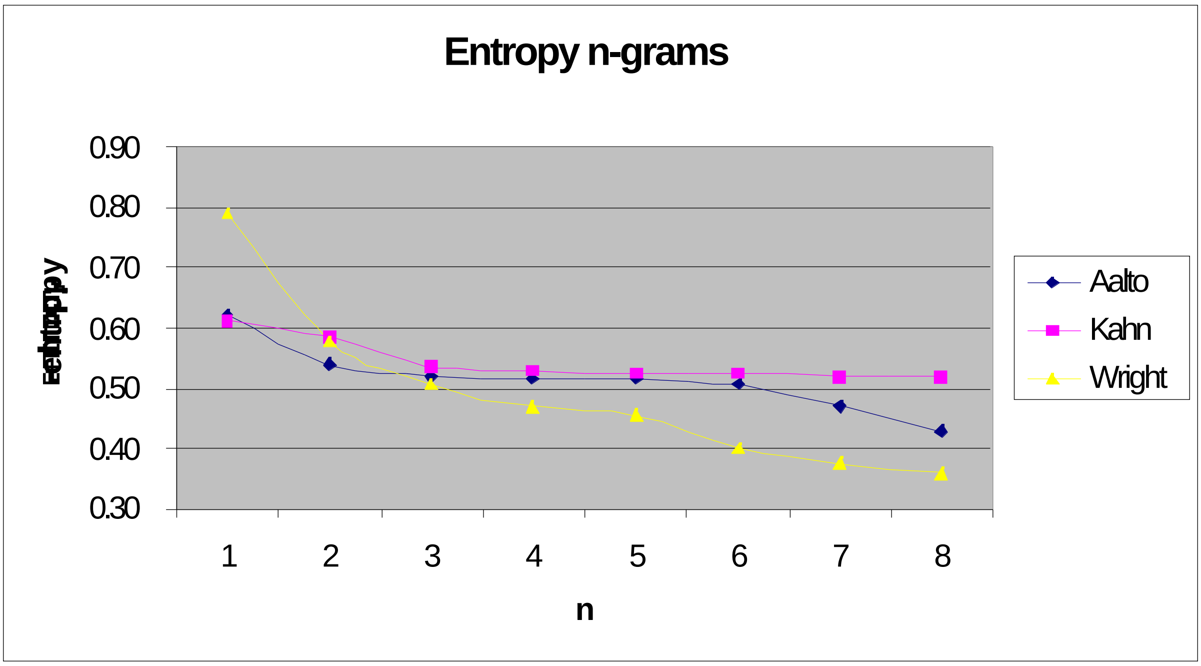


Fig. 2: The n-gram of relative entropies of angle-based Q-codes for Aalto's and Kahn's architectural plans for houses and for Wright's prairie house plans.

Figure 2 shows that for Aalto and Kahn the entropy drops at approximately the same rate, which is much slower than the rate for Wright. The drop rate of entropy can be interpreted as the average amount of information carried by one character in the n-gram (Stalling, 1998). Therefore the information in Figure 2 can be interpreted as an indication that the complexity (the influence of long Q-code clusters) for Aalto and Kahn house plans is approximately the same and higher than that the complexity of Wright's prairie house plans.

Conclusions

What we have been able to show is that it is possible to "measure" drawings. We constructed a measurement based on the information content (in information theoretic terms) of the drawing. We implemented this using a qualitative representation of drawings and applied it to the outlines of architectural plans. What we able to show was that it is possible to measure the complexity of drwaings and to use that measure to discriminate between the bodies of drawings of different designers designing for the same class of requirements.

Architectural designs are often driven by aesthetic considerations as much as functional requirements, however, we were able to effectively measure the complexity of architectural drawings.

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References

Gero, J. S. and S-H. Park (1997), "Computable feature-based qualitative modeling of shape and space", *in* R Junge (ed.), *CAADFutures 1997*, Kluwer, Dordrecht, 821-830.

Goldschmidt, G. (1994), "On visual design thinking: the vis kids of architecture", *Design Studies*, **15**(2), 158-174.

Pincus, S. (1991), "Approximate entropy as a measure of systems complexity", *Proceedings of the National Academy of Sciences*, **88**, 2297-2301.

Purcell, T. and J. S. Gero,(1998), "Drawings and the design process: A review of protocol studies in design and other disciplines and related research in cognitive psychology", *Design Studies*, **19**(4), 389-430.

Shannon, C (1948), "A mathematical theory of communications", *Bell Systems Tech J*, **27**, 379-423.

Stalling, D. J. (1998), "Understanding the second-order entropies of Voynich text," http://champollion.nu/english/voynich/mbpaper.htm

Suwa, M. , J. S. Gero and T. Purcell (1999), "Unexpected discoveries: How designers discover hidden features in sketches", *in* Gero, J. S. and Tversky, B. (eds), *Visual and Spatial Reasoning in Design*, Key Centre of Design Computing and Cognition, University of Sydney, Sydney, Australia, 145-162.