

The topology of social influence and the dynamics of design product adoption

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This paper presents the results of studies on how the dynamics of design product adoption is affected by the topological structure of social communication and influence between consumers, without any changes in the designed product. The dynamics of product adoption are studied over random, modular small world, and scale free social network structures, under local rules of communication. Results show global behaviors emerging from these local agent communication rules, including states where populations completely accept or reject products, starting from similar initial states, as well as regimes and cycles of synchronized behaviors of adoption and rejection. It is claimed that without modeling consumer interactions, understanding and modeling of innovation in design will remain inadequate. Since there could be fundamental limitations to the predictability of adoption behaviors under social influence, innovation strategies could fare better when they focus on the quality, novelty, and technological advances they could bring instead of being guided only by social popularity and influence.

Introduction

Traditionally design research has focused on the design processes of the individual designer [1-3]. This was followed by research into the design behavior of teams of designers, whether collocated or remotely located [4-6]. More recently the product's consumer has been brought within the ambit of product design and innovation through feedback of consumer behav-

ior and its effect on product design cycles. Information about consumer behavior is becoming increasingly available through their use of social media as they search for and decide to consume products [7,8]. As a consequence the consumers' behavior is increasingly taken into account as part of an expanded view of the design activity.

Designing, then, ceases to be a linear activity in which an individual or group of producers generate products, deliver it to consumers, who then consume the product. It also ceases to be an *iterative feedback based cyclic activity* in which the feedback from consumers is used to improve or innovate on products for the next cycle of generation. Instead, designing becomes a *networked social activity*, where the dynamics of social communication between the actors (producers and consumers) governs whether or not and how a product is adopted.

It has been shown in live Web based social experiments, for example, that with the quality of the products remaining exactly the same, the presence, absence and degree of social communication between agents could lead to completely different regimes of adoption [9]. While product quality did matter, in that while the best products rarely did poorly and the worst ones nearly always suffered, but with a large margin, increasing the strength of social influence amongst consumers made the rankings both more unequal and unpredictable. Thus, predicting exactly how successful a product will be, and how a producer must change their innovation or production strategy becomes a particularly hard problem to solve, given that even chance communication on social networks could change the outcomes dramatically, with no change in the profile of products competing in the market.

Further, social influence may or may not always be positive. For example, the networked structures of technological innovation diffusion that are too deeply influenced by a majority supported social communication could actually end up limiting, hindering, or slowing down innovation [10]. Thus, to understand the scenarios under which innovations succeed or fail, it will be important to understand the structure of topological social interactions between consumers, even if one possible emerging policy from such studies is to weight personal values and opinions more than the prevailing social opinion in specific cases in order to meaningfully advance innovation.

As part of a larger scale project that covers the interactions between designers, the interactions between designers and consumers and the interactions between consumers, in this paper we present a preliminary effort to model the diffusion of social influence by consumers over social networks and study how this affects the dynamics of product adoption over time. In

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particular, under simple and standard local rules of social interactions between consumers, we empirically study whether the topology of the social network has any significant effect on global adoption behaviors for the entire population of consumers.

We study whether under a specific network structure, there is any certainty of a product failing or succeeding, without changing its design quality. We note here that in this preliminary model, we consider design quality and other factors as constant. This allows us to focus on a simple model of social communication and its effect on adoption. Since (real) adoption behaviors are dependent on both design quality as well as social communication, along with host of other factors such as production and distribution strategies, marketing techniques, and demographics, we believe it will be useful to focus on a simple model where all else is held constant. This allows us to specifically focus on the question of social communication in order to understand whether the structure of the network on which social communication occurs has any significant role in the set of emerging behaviors.

This research is part of a larger project that treats designing as an activity that includes the social behavior of both designers and consumers as groups and their interactions within each group and across each group through designed products within its ambit. Social behavior involves both interactions and the consequential change of the values of designers and consumers. It is claimed that without modeling such interactions it is not possible to adequately model innovation. We find that there could be fundamental limitations to the predictability of adoption behaviors under social influence, innovation strategies could fare better when they focus on the quality, novelty, and technological advances they could bring instead of being guided by social popularity and influence. Since it has been observed in the Salganik study that good products rarely do poorly [9], but there is a sufficient level of uncertainty added on account of social influence, the state of the design field could progress faster when innovation or advancement of quality becomes a major focus (reflecting ultimately in the quality of the product, making it one of those good products that would rarely fail).

However, the inherent unpredictability of product adoption leads also to a more counter-intuitive observation: if, holding design quality constant, adoption behaviors are fundamentally unpredictable, then a lazy designer or producer could simply aim to produce many below standard products rather than a few well designed products, in the hopes that at least some of them would succeed purely by contingency. This more counter-intuitive behavior is also empirically observed. For example, in the case of online

products such as songs and dances, where people post multiple videos (e.g. on YouTube) that are not of sufficiently high artistic quality, but very large in number. The hope is that at least one of them will become “viral” purely by contingency, making the artist famous, after which even sub-standard work will continue to be adopted. However, we argue, that while such behavior may ensure a short-term advantage to the producer, it may actually hinder real innovation or advancement of the design field.

We model random, small world modular (clustered), and scale free networks of social relationships between consumers, with consumers who could either be *excitatory* (send out positive influence about the product), or *inhibitory* (send out negative influence about the product) for the population of consumers. Each consumer has a *state*, that is, *adopt* or *reject*. This state is a decision for the consumer to make at each time step as socially connected neighbors send out influence messages. Then, we study the dynamics that evolve over several time steps as consumers send out messages to their neighbors. Each consumer receives a positive or negative influence from its neighbors, which helps to inform their state, but the consumer also listens to “itself”, that is, its own recommendation. While the current system has no history, the same framework could incorporate history of use as a factor informing the consumer’s decision for the next time step in future work.

We report several expected as well as unexpected results on the dynamics, their attractor states, and implications for product adoption behaviors that can emerge out of social communication over networks.

Significance

In its most general form, the problem addressed in this paper has historically been addressed in both biology and the physical sciences [11,12], as well as social sciences and economics [13-16]. It could be described as the problem of predicting how a system’s elements would behave as a collective when local information transfer occurs between the elements. Not all approaches have adopted network based analyses, (for example, game theory and information theory have been widely employed to study the problem), and the network based modeling approach shows the fundamental importance that networks of social communication play in deciding consensus emergence, information cascades, or targeting and identification of most influential customers to maximize adoption of innovations and products.

Here we show empirically that while different types of social network structures have different forms of adoption behaviors, (modular and clus-

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tered networks, for example, could enter into regimes where groups of customers collectively adopt or reject products, and sparse random networks could demonstrate synchronized or cyclic behaviors of adoption and rejection). There is an inherent unpredictability associated with the dynamics even with all network parameters remaining constant. While the model presented here is abstract, and we do not treat the properties of the model analytically, or make any claims or comparisons about the empirical similarity of the model's behavior with real world regimes (which is out of scope for this first preliminary paper), the inherent unpredictability of dynamics suggests an interesting, albeit debatable, finding about innovation strategies by designers. This is to do with the decision on the degree to which social popularity and influence should be used as a guiding basis for driving innovations. If there is inherent unpredictability of dynamics in the system, with system structure, initial conditions, and the parameters remaining constant, an innovation could fare better by focusing on the quality, novelty, and technological advances that it could bring instead of being guided only by social popularity and influence. That is, as discussed above, focusing on design quality lends a higher chance of advancing the design field as well as success of adoption, whereas the strategy of many sub-standard products being introduced in hope of success, could lead to short term gains, but not necessarily advancement of the field.

Method

A network of consumers is defined as a graph $G = (V, E)$, where $|V| = n$ is the number of consumers, and E is a set of edges connecting two consumers i and j from V . The graph G is represented as an $n \times n$ matrix A , where $A_{ij} = 1$ if consumers i and j are connected, and $A_{ij} = 0$ otherwise. The set E could be constructed in several ways, which will affect the resulting social network structure of G . Here we consider three such models: random, small world modular, and scale free connectivity. We note that the construction of the social network could be motivated from several bases, but a common one is patterns of co-usage of a product. That is, a link exists between consumers i and j if they have used common products before.

Random connectivity between consumers

Random connectivity between consumers is modeled as an Erdos-Renyi (ER) random network, where a link exists between each pair of consumers

with an independent probability of p . That is $G_r = (V, E_r)$, with A_r representing the adjacency matrix for the random graph. While p can take any value from 0 to 1, we choose p to be low, as most product customer usage networks are very sparse. The lowest values of p could be chosen based on the criteria that a value of $p > \frac{2 \ln n}{n}$ would produce a connected graph. A p lower than this limit would likely produce disconnected components. We will consider values of p above and below this limit.

Under conditions of random connectivity, a consumer is equally likely to receive social messages from any other customer in the population.

Small world modular connectivity between consumers

Different from random connectivity is the idea of small world modular connectivity. Modular social networks capture the idea that consumers are likely to have tightly clustered social interaction groups, where a group of customers could be using a product and passing recommendations to only a small part of the entire population with large probability, and to the rest of the network with lower probability. This models the situation where people may recommend to close friends or other consumers similar to themselves more than they do to distant friends and acquaintances or other dissimilar consumers in different social groups.

It has been shown that modular networks are likely to be small world [17]. That is, modular networks will have the properties of high clustering coefficients and low average path lengths between nodes. A modular network is modeled using a stochastic block model [18-20]. A modular network is modeled as a graph $G_m = (V, E_m)$, where the set of edges E_m represents the links between two nodes from V . The graph is represented by its adjacency matrix A_m , and has a special structure to represent modularity. Let there be q communities of size z nodes each, such that the number of communities $M = n/z$. Thus, each node has a label from the set $\{1, 2, \dots, q\}$. If two nodes i and j have the same label, then they are connected with a probability of p_{in} , and if they have different labels, they are connected with a probability of p_{out} . Since the probability of connection within a module is higher than that between modules, we have $p_{in} > p_{out}$. Further, we model sparse networks with the total number of edges scaling as $O(n)$, since most real world social networks have sparse connectivity.

In a modular network, each consumer has a higher probability of communicating with members of its own group than members of other groups.

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Scale free connectivity between consumers

Another basis of connectivity between consumers could arise from the idea of influential consumers in the network, that is consumers who are enthusiastic adopters, and therefore use lots of products, and consequently are connected to lots of other consumers through commonality of product use. We model a situation where there are a low number of these influential well connected customers, amongst many more consumers who are not as well connected (i.e., the not so well connected consumers use few products, and therefore are connected to only a few others). Since this network is a mix of a low number of well connected and a high number of not so well connected consumers, it is likely that each well connected consumer will be connected to lots of other consumers themselves who will have low connectivity. The ideal situation in this condition would be a “star” network with one consumer in the middle, connected to and recommending a product all the other consumers in the network.

But, to model a more realistic situation with the above properties, we use the Barabasi-Albert scale free network structure model [21]. The scale free connectivity network is modeled by a graph $G_s = (V, E_s)$, where E_s represents the set edges between any two consumers from V . In this model, the network is built in successive time steps, where each new incoming node j is connected to an existing node i with a probability p_s proportional to the existing degree of node i . That is, a “rich get richer” model, where nodes with high connectivity keep receiving a high proportion of the connectivity from incoming nodes, resulting in a situation where the network has a low number of highly connected nodes and a high number of nodes with low connectivity, at each stage (hence the name scale free, since recursively removing all the top degree nodes, one would again see the same scale free structure of degree distribution).

A scale free network models the situation where a low number of very influential customers are able to influence and send recommendations to many customers who are “influenced”.

A simple model of dynamics of social communication

On each of the network types described above, each consumer has a state s , with $s = +1$, if a consumer has adopted a product, $s = -1$, if they reject the product, and $s = 0$, if they are in an undecided state (equally likely to adopt or reject). Further, the current state of each consumer is the social

message that is passed on to other neighbor consumers, but all the four following possibilities could exist:

1. Consumer i uses product ($s_i = 1$) and sends out an excitatory positive recommendation to its neighbor j ($A_{ij} = +1$).
2. Consumer i uses product ($s_i = 1$) and sends out an inhibitory negative recommendation to its neighbor j ($A_{ij} = -1$).
3. Consumer i does not use or rejects product ($s_i = 0$) or ($s_i = -1$) but sends out an excitatory positive recommendation to its neighbor j ($A_{ij} = +1$).
4. Consumer i does not use product ($s_i = 0$) or ($s_i = -1$) but sends out an inhibitory negative recommendation to its neighbor j ($A_{ij} = -1$).

Thus, we have a structure for A where an entire column could be positive or negative (where a link exists), depending on whether the consumer is sending out excitatory (positive) or inhibitory (negative) recommendations to others. The proportion of excitatory and inhibitory customers could then be varied as a parameter to study the dynamics.

The networks are initialized with a $1 \times n$ state vector S , where the n components represent the initial states of each of the n customers, and could be initialized randomly to either +1 or -1 with equal probability. This decision is not trivial and deciding how many and which customers to target in order to maximize influence is a defined problem [16], but we fix this for the purpose of this particular paper since we will look at the dynamics under different types of network structures. Then, based on the local rules above, similar to [11-12], and a pre-decided proportion of excitatory and inhibitory agents and choosing a particular network connectivity type, the state of each customer at time step $t + 1$ is decided by the inputs it receives from other customers about their states at time t and their recommendations as:

$$S_i(t + 1) = \text{sgn} \left(\sum_{j=1}^n A_{ij} S_j(t) \right),$$

where the sgn function represents the sign function with $\text{sgn}(c) = 1$ if $c > 0$, $\text{sgn}(c) = 0$ if $c = 0$, and $\text{sgn}(c) = -1$ if $c < 0$. Note that here the agent also receives its own recommendation from the previous time state, since we assume that the self-link A_{ii} is 1 or -1 depending upon whether the customer i is an excitatory or inhibitory customer.

Starting from an initial assignment of S , we let the dynamics run till the whole network stabilizes to a stable point or a limit cycle. We present the results of these runs in the Results section.

Results: Random connectivity dynamics

In this section the dynamics of product adoption under random social communication conditions is reported. For the experiments, we varied network sizes n from a few hundred to a few thousand nodes and varied the connection probability p of the random network from below the limit $2 \ln n/n$ (producing disconnected components) to above the limit $2 \ln n/n$, (producing a connected component). When the connection probability is higher than the limit, i.e., the graph is a connected network, the networks settle down to states of either complete adoption or complete rejection by the entire population of consumers. Figures 1(a) and (b) show one state of complete rejection and one of complete adoption, starting from the same initial starting states of product assignment. That is, at time step 1, states S for each customer are assigned, in which each customer is equally likely adopt (+1) or reject (-1) a product. As the model of dynamics of social recommendations discussed in the previous section is allowed to unfold, it is observed that for different runs the entire network either quickly adopts or quickly rejects the product. Once the stable state is reached, it is unchanged for the future.

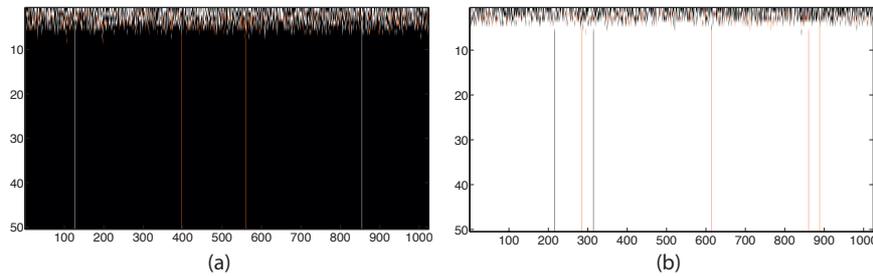


Fig1. Connected random social network communication dynamics. (a) $n = 1024, p = 0.014$ (just above the limit $2 \ln n/n$). (b) $n = 1024, p = 0.02$ (above the limit $2 \ln n/n$). The rows show the time steps of dynamics evolution, the columns show consumer ids. States are represented by color, Black = reject (-1), White = Adopt (+1), Red = 0 (undecided).

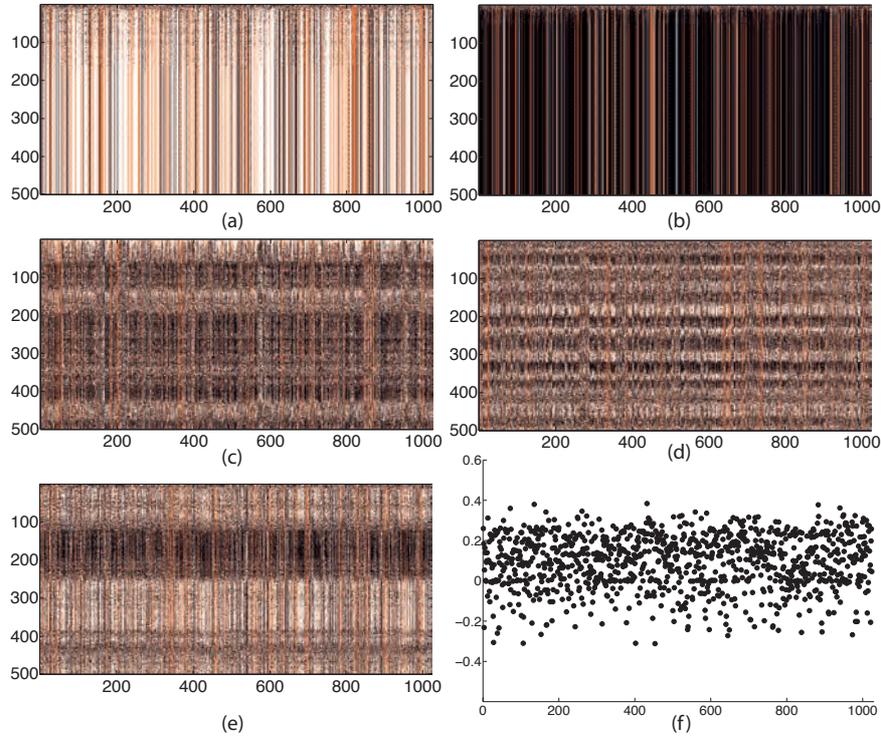


Fig2. Disconnected components random social network communication dynamics, $n = 1024, p = 0.003$ (below the limit $2 \ln n/n$). (a) Most consumers adopt the product (b) Most consumers reject the product (c) Bands of synchronized adoption and rejection behaviors through time. The rows show the time steps of dynamics evolution, the columns show consumer ids. States are represented by color, Black = reject (-1), White = Adopt (+1), Red = 0 (undecided). (f) A plot of the average activity of each agent over all the time steps from (e).

Results: Modular connectivity dynamics

In this section the dynamics of product adoption under sparse modular social communication conditions is presented. Figure 3 shows examples of the dynamic behavior that unfolds. Once again, with similar initial state assignments S , two final states of near complete adoption and near complete rejection are observed in a few runs; i.e., a final state of in which either all modules adopt the project or reject the product. However, it is also seen that different modules can reach different stable states, with a few modules in states of adoption co-existing with other modules in states of rejection.

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Particularly interesting is the behavior of a few individual customers who cycle between the combinations adopt-undecided, adopt-reject, and reject-undecided. Another interesting observation is that the primary intra-module global behavior can continue for long spans of time, but can suddenly switch to another state, Figure 3(d). The mean activity of consumers over 100 time steps in Fig. 3(e) is shown in Fig. 3(f), where the difference between global behaviors in different modules is clearly observed.

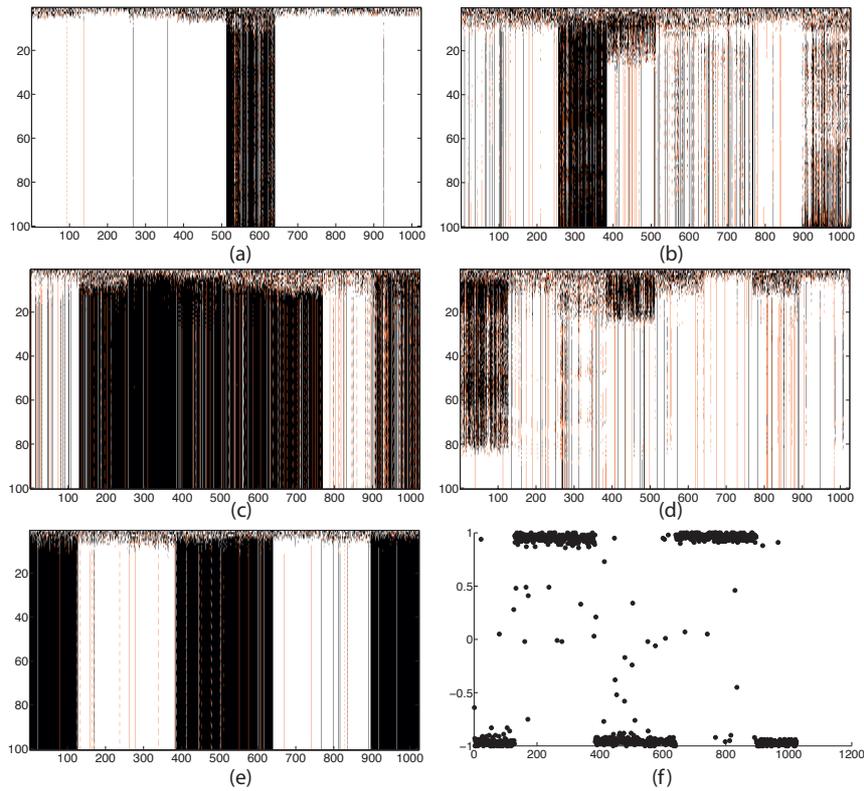


Fig3. Modular social network communication dynamics, $n = 1024$, $p_{in} = 0.1$ to 0.005 , $p_{out} = 0.005$ to 0.001 , results from several combinations are shown and discussed in the main text. (a)-(e) Dynamics of modular adoption and rejection behavior over time, some modules completely reject while some others completely adopt the product. The rows show the time steps of dynamics evolution, the columns show consumer ids. States are represented by color, Black = reject (-1), White = Adopt (+1), Red = 0 (undecided). (f) A plot of the average activity of each agent over all the time steps from (e).

The probability of intra and inter modular connectivities, p_{in} and p_{out} have a similar unexpected effect as observed for the random case: more complex behavior is seen for sparser cases. When p_{in} is high, for example, $p_{in} = 0.1, p_{out} = 0.005$ in Fig. 3(e), leading to denser networks, the most common outcomes are consumers confirming strongly to the global behavior of their module. Adoption is strong, but so is rejection, and once a state of rejection sets in it is impossible to change it to adoption. However, when p_{in} is low [for example, $p_{in} = 0.05, p_{out} = 0.001$ in Figs 3(a)-(d)], leading to sparser networks, outcomes are more unpredictable, rich and varied. This is unexpected, because it shows richer product adoption dynamics occur when networks are sparser: long spans of rejection could still flip and show regimes of adoption behavior.

Results: Scale free connectivity dynamics

In this section the dynamics of product adoption under scale free social communication conditions is reported. While in the modular case entire modules or groups of consumers show similar behaviors, in the scale free case, the global behavior is more individual and governed by each consumer's connectivity to highly connected influential consumers.

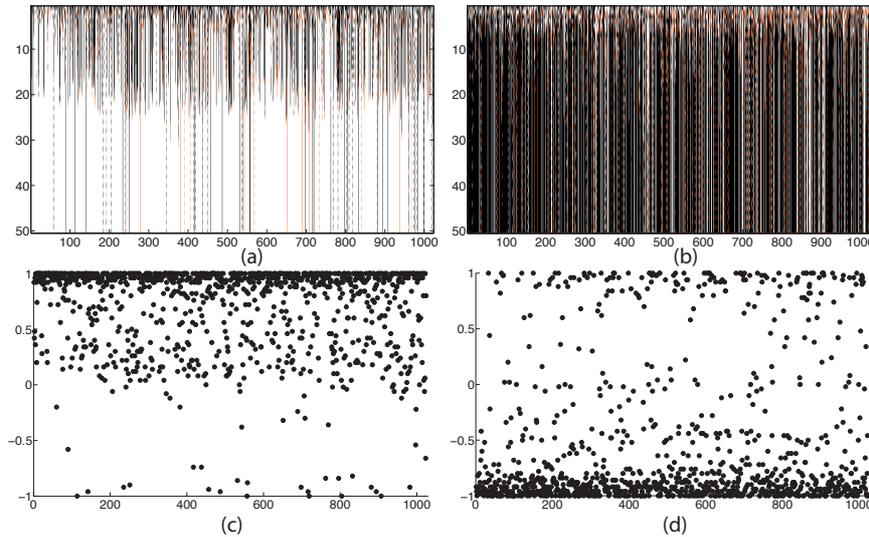


Fig4. Modular social network communication dynamics, $n = 1024, p_{in} = 0.1$ to $0.005, p_{out} = 0.005$ to 0.001 , results from several combinations are shown and discussed in the main text. (a)-(e) Dynamics of modular adoption and rejection behavior over time. The rows show the time steps

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of dynamics evolution, the columns show consumer ids. States are represented by color, Black = reject (-1), White = Adopt (+1), Red = 0 (undecided). (f) A plot of the average activity of each agent over all the time steps from (e).

In this case, an interesting result emerges, where starting from the same initial state assignments S , a majority of consumers in the population settle to a stable adoption or rejection state, but a few customers show persistent strong behaviors that are opposite to the main population behavior. Figures 4(a) and (b) show a primarily adopting and primarily rejecting population, but a few individuals (not negligible in number) continue to show the opposite stable behavior. Figures 4(c) and (d) show the corresponding mean activity states for each consumer in (a) and (b) respectively.

Similar to the modular case, a significant number of consumers also show cyclical behaviors, switching between the combinations adopt-undecided, adopt-reject, and reject-undecided. This cyclical behavior emerges as stable, continuing indefinitely. Further, the patterns of the cycles (the numbers of time steps over which one state lasts followed by another) show a rich diversity.

Conclusions

In this paper, we modeled the diffusion of social influence by consumers over social networks and studied how this affects the dynamics of product adoption over time. In particular, under simple and standard local rules of social interactions between consumers, we empirically studied whether the topology of the social network has any significant effect on global adoption behaviors for the entire population of consumers.

Using a simple model of social influence, the dynamics of product adoption are studied over random, modular small world, and scale free social network structures, under local rules of communication. Results show global behaviors emerging from these local agent communication rules, including states where populations completely accept or reject products, starting from similar initial states, as well as regimes and cycles of synchronized behaviors of adoption and rejection. Even with very simple social communication structures, inherently unpredictable complex global behaviors of agents emerged, and the nature of the structure of social communication played an extremely significant role even with the structure or attributes of a designed product remaining unchanged and constant. An analytical assessment of the model was out of scope for this paper, but in future work, we will study both the model the different regimes of results analytically. In future work we will extend the model with data from real world recommendation systems and product adoption records, in order

to test the findings and make the abstract model presented here closer to empirical observations of real product adoption and social communication over networks.

In particular, we observe that with unchanged parameters and initial states, different runs produce a rich variety of behaviors, suggesting that any pattern of adoptions and rejections is possible. We also observed that denser networks were rigid and necessitated quick convergence to stable behaviors for entire populations, whereas sparser networks demonstrated much richer diversity of outcomes. Without modeling such interactions, understanding and modeling of innovation in design would remain inadequate. Since there appear to be fundamental limitations to the predictability of adoption behaviors under social influence, in order to advance the design field, innovation strategies could fare better when they focus on the quality, novelty, and technological advances they could bring instead of being heavily guided by fragile collective behavior, social popularity and influence.

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