Shape Pattern Representation for Design Computation

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Abstract. Properties of shape pattern schemas are investigated to represent shape patterns as formative ideas for supporting design computation. Shape pattern knowledge is identified from not only physical shape primitives but also spatial relationships. Using these notions, properties of shapes and shape schema, such as subshapes and group shapes, shape similarity, variability and embeddedness are explored, and representations of shape patterns in designed objects are presented.

1 Introduction

Previous design knowledge learned and abstracted from existing artefacts plays important roles in design procedures. It works as a source for the derivation of designs, controls and guides the design process and allows the designer and the computer to evaluate the results of designing. For example, knowledge of symmetry and proportion that Vitruvius and others discovered from human bodies and ancient buildings has produced important design principles in architectural design. Many aesthetic values have been evaluated using proportion and symmetry (Birkhoff, 1932). One aspect of design knowledge which appears in architectural drawings is the way shapes are organised: a shape pattern. A distinct and replicate syntax or compositional relationship between shape elements is analysed formally and regarded as a shape pattern. It is an invariant in shape objects that appears repeatedly in one object or in a set of objects.

Recognition of this design knowledge from existing objects and its representation are important for supporting computer-based design procedures as well as for human design activity. Shape patterns are recognised in terms of repetitions of similarities as well as repetitions of the same relationships or shapes. A shape pattern, that is a repetition of shapes, such as a repetition of the same triangles, a repetition of red rectangles, and a repetition of ovals, can be easily recognised using feature similarity. However, a pattern that is constructed from different low-level shapes in a complex shape is not recognised in terms of feature similarity. Similarity of spatial relationships needs to be considered for shape pattern recognition and representation. A shape pattern representation that describes spatial relationships as well as physical shapes has been developed. According to Brachman and Levesque (1985), knowledge needs appropriate representation through some languages or communication mediums that correspond in some salient way to the world or a state of the world to allow a machine to manipulate the knowledge and come to new conclusions. Also, these authors suggest that any knowledge representation should satisfy certain conditions, such as expressive adequacy, reasoning, efficiency, primitiveness, meta-representation, incompleteness and real-world knowledge. A pattern is represented with symbols and numbers based on a schema that represents the generic concept upon which all information processing depends (Minsky, 1975; Schank and Albelson 1977; Rumelhart, 1980). A shape pattern that is recognised and represented operationally is useful for design procedures such as style learning, shape analogy and shape complexity measure.

The objective of this paper is to provide shape pattern representations suitable for supporting design processes. Recognition of spatial relationships as well as physical shapes from complex shapes has been studied and their complexity and properties are investigated in order to represent them. Schema representations of shapes and their relationships are explored in terms of similarity. Finally, this paper discusses some possible applications of shape pattern representation for design computation. The domain of design ideas lies within the formal and spatial realm of architecture, thus the political, economic and technical aspects of architecture are excluded.

2 Shape patterns as design knowledge

From pre-history, humans have decorated fabrics, pottery vessels, tools and buildings with patterns. Repeated usage of the same materials in decoration gives artefacts patterns. The craftsmen's understandings of materials create good patterns. Natural orders surrounding us provide examples of patterns. Forms of plants are the most popular motifs in capitol designs, window designs and many other artefact designs. Patterns have many characteristics in design. They often encapsulate design knowledge which appears in the existing design. Patterns are generalised from a set of objects belonging to a class. Related patterns can be composed in a hierarchical form and layered. Patterns expressed in the form of hierarchical tree structures have variability at their lower levels. The application of invariant high-level patterns can generate many possible results under different contexts.

In complex objects, there may be many independent specific sets of sub-elements that are repeated and arranged in certain ways. Patterns are representations of these small blocks and clearly identify a synthesis of primitives. They help designers and computers to understand and interpret the design. Patterns as small wholes are recognised and work as parts for the overall whole. Lower level patterns that are hierarchically formed can be turned into variables. In pattern recognition, a matching process that identifies the relationships among patterns disregards micro-patterns (or low-level patterns). This variability remains within the borders of higher-level constraints.

Patterns that are good solutions for certain problems and contexts may be applied elsewhere. The flexible application of one pattern for different problems and situations can proceed by variable instantiation and analogy. Instantiation of pattern variables as well as physical shape variables generate design objects that belong to a class specified by the high-level patterns. Patterns represent abstract formal knowledge for objects, thus it is possible to transfer patterns from one design domain to another.

2.1 Formative shape pattern

A pattern is a design in which a certain shape is repeated many times (Rowland, 1964). In this paper, similar spatial relationships as well as similar shapes that are recursively arranged are considered as a shape pattern. These are constructed in the form of a hierarchical tree structure as a shape schema. A pattern is knowledge generalised from a class of objects so that it captures the essential characteristics of groups of individual shapes. For example, a graded transition of spaces characterises much of Aalto’s architecture in Figure 1.

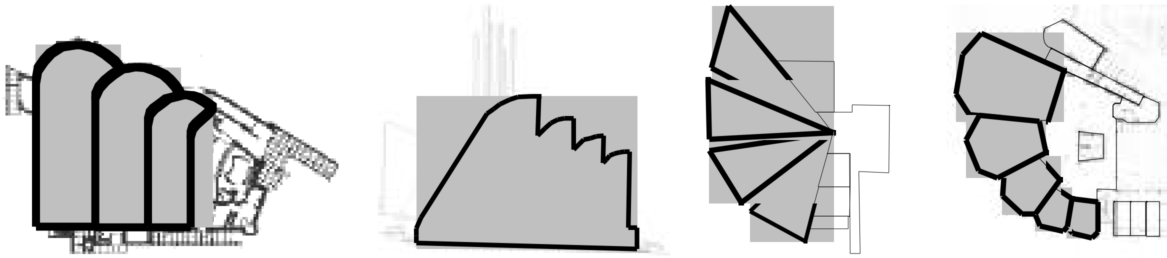
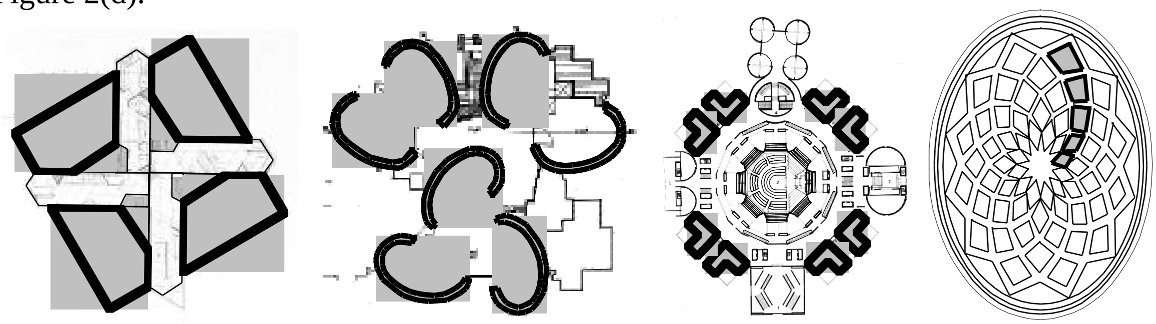


Figure 1. Formative shape pattern in Aalto’s architectural designs.

Similar shapes may appear in an object or a class of objects repeatedly and they are arranged in certain ways. A set of repeated similar shape elements specifies a configuration. A structured configuration is considered as a formative shape pattern. Sets of similar shapes in Figures 2(a) and 2(b) are repeated and produce patterns. Furthermore, similar patterns may be repeated in a set of objects with certain relationships, and identify a high-level pattern recursively. Reflection relationships of two L-shapes are repeated in Figure 2(c) and arranged in a rotational pattern. Sets of quadrilaterals that identify gradation of sizes are repeated in Figure 2(d).



(a) (b) (c) (d)

Figure 2. Repetition of similar shapes and shape patterns.

2.2 Formative ideas

According to Durand, there are two kinds of design elements in architectural buildings; architectural elements and geometric elements. Architectural elements are functional elements that can be found in most buildings such as walls, columns, openings, roofs. A building is the result of combination of the architectural elements. The inner structure of these architectural elements could be a formative idea. Geometric elements are simplified shapes from physical elements represented by lines and points, which are abstract and conceptual. A formative idea is the way of composing these elements. It can be formed through a process of reducing a complex of formal variants to a common root form. It is understood as the inner structure of a form or a concept concerning the form. Thus a formative idea is organised in terms of relationships and contains the possibility of infinite formal variation and further generation. It helps designers to organise decisions and provides formal order. It is useful for conscious generation of forms. The great buildings in history share many formative ideas, such as linear symmetry, balance, gradation, rotational symmetry. Formative ideas can be abstracted and generated from many sources: existing designed shapes, natural shapes or metaphysical notions. Vitruvius suggested that the structural form of a wooden house gave the form for the stone temple. Also many designers abstract formative ideas from natural shapes. For example, a spiral shape is formalised from a trumpet shell. Some metaphysical notions can be formalised. For example central plans of temples are seen to symbolise expressions of unity. Figure 3 shows examples of formative ideas, their representations are presented later in this paper.

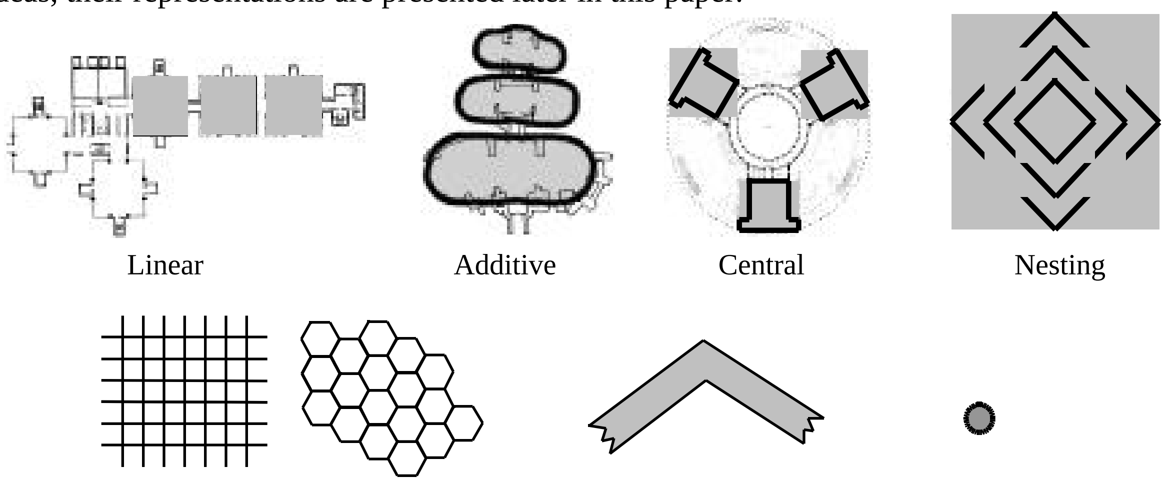


Figure 3. Formative ideas.

3 Shape Representation

There is a sense in which every design computation contains knowledge about problems, solution procedures and results. It is necessary to represent this knowledge explicitly. The power of design computation lies in the explicit representation of knowledge that the computer can access. One reason for knowledge representation is that new conclusions can be arrived at by manipulating the representation.

Adequacy of the representation language is critically important, because the process of shape pattern representation can be regarded as a search for plausible descriptions that the computer formulates from given input data. Firstly, the representation language should describe the particular concept appropriately and it should have the ability to represent implicit as well as explicit knowledge. Secondly, it should have a well-organised form so that it can perform inferences correctly with predefined knowledge. Thirdly, from the result of shape pattern representations, new objects should be able to be predicted or generalised.

Considering these requirements, predicate calculus in a hierarchical structure as a schema is employed and developed as the shape pattern representation language, because of its well-defined syntax and semantics. It is composed of predicates and arguments. Predicates specify the types of relationships, and arguments give the details.

3.1 Some existing shape representations

Shapes and their representation have been studied in many areas of design computation, such developments include shape grammars and architectural morphology (Stiny, 1975, 1976, 1978, 1980; Steadman, 1983).

Shape grammars were developed by Stiny and Gips (1975). In shape grammars, shapes are manipulated in terms of rules to generate shape designs. Shapes are composed of lines and may be labelled with symbols to allow recursive rule applications. Rules are transformations of one shape to another that allow parts of shapes to be defined and changed recursively to conform to given spatial relationships. Shape grammars can describe ways of combining shapes. Applications of the same vocabularies and generating rules can produce a class of designs in a particular style (Stiny and Mitchell, 1978; Knight, 1980; Koning and Eizenberg, 1981). It is a good tool to describe a language of design. However, it describes them linearly in one layer. Most design procedures and languages are hierarchically formed and multi-layered. More sophisticated ways of describing shape designs need to be developed.

Architectural morphology has been developed by a number of researchers including Steadman (1983) with the notion of a general science of possible forms to describe syntactic arrangement of all possible plans using graph theory. Architectural plans can be represented with nodes and arcs, nodes denote rooms or spaces and arcs specify adjacencies between them. It can describe the structure of three-dimensional as well as two-dimensional arrangements of spaces. But its spaces denoted by nodes are limited to simple shapes, and its relationships deal with only adjacency or accessibility rather than other spatial relationships.

3.2 Shape definition and representation

Shape: A shape is composed of subshapes, and shapes may have relationships with each other. Shapes are recognised explicitly and implicitly. A shape that is initially represented explicitly is a *primary shape*, and a shape that exists only implicitly in a primary shape is an *emergent shape* (Gero and Yan, 1994; Gero and Jun, 1998). Among those shapes, *bounded polyline shapes*[1] are considered as shapes in this paper.

An architectural drawing is a set of bounded polyline shapes represented as follows:

$$S = \{P_1, P_2, \dots , P_n\} \qquad (1)$$

Where P: bounded polyline shape
S: shape object

There are many different shapes in terms of shape properties: subshape, primitive shape and group shape.

Subshape: A shape can be decomposed into parts. Parts are subshapes embedded in a shape and they may have spatial relationships between each other, Figure 4(a). Stiny (1980) defines the subshape as follows:

> " One shape is a subshape (part) of another shape whenever every line of the first shape is also a line of the second shape. More precisely, a line is in a shape if and only if its end points are coincident with a maximal line of the shape. Thus, a shape

[1] A bounded polyline shape is an enclosed polyline shape, for any point on the boundary of which there exists at least one circuit composed of line segments which starts from and ends at that point without covering any line

S_1 is a subshape of a shape S_2 (denoted by $S_1 \le S_2$) if and only if each maximal line of S_1 is in S_2."

Primitive shape: A shape is composed of a set of subshapes and their relationships (Stiny, 1976). Relationships are compositional rules of shapes in shape grouping. Shapes that cannot be decomposed into subshapes are primitive shapes, among these, squares, circles and equilateral triangles are regarded as pure primitive shapes. In addition, shapes that can be divided into subshapes but whose bounded polyline shape is more important than the subshapes are considered as primitive shapes in the construction of shape knowledge. In Figure 4(b), the square can be decomposed into triangles, but sometimes its subshapes may be discarded and the bounded polyline that is a square is considered as a primitive.

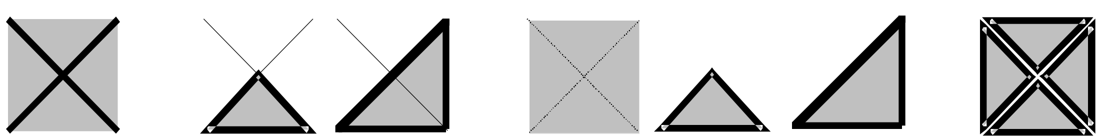
Group shape: Shapes are grouped together visually in terms of their properties and relationships, such as proximity, similarity, closure, good continuation, symmetry (Arnheim, 1954; Kohler, 1930; Wertheimer, 1945). Congruent primitive shapes are grouped together and form a group shape based on Gestalt laws. Also some congruent group shapes are composed into a higher group shape. Subshapes, either primitive shapes or group shapes, can be embedded into other subshapes. The group shape may be regarded as a primitive shape when a set of congruent group shapes are grouped together to make a higher-level group shape. Subshapes are congruent primitive shapes or congruent group shapes, Figure 4(c). A shape object is composed of group shapes and it is represented as follows:

$$G_i = \{P_{i,1}, P_{i,2}, \ldots , P_{i,n}\} \quad (2)$$

$$S = \{G_1, G_2, \ldots , G_n\} \quad (3)$$

Where G: group shape
S: shape object

Figure 4 shows the difference between a primitive and a group shape even when they are the same shapes. The square with diagonal lines as a primitive in Figure 4(b) can be recognised as a square, the diagonal lines are disregarded. But in Figure 4(c), the shape can be decomposed into four triangles and each shape has a 90^o rotation relationship with adjacent shapes. One shape can be represented and interpreted in various ways.



(a) Subshape (b) Primitive shape (c) Group shape

Figure 4. Subshape, primitive shape and group shape.

3.3 Spatial relationships and representation

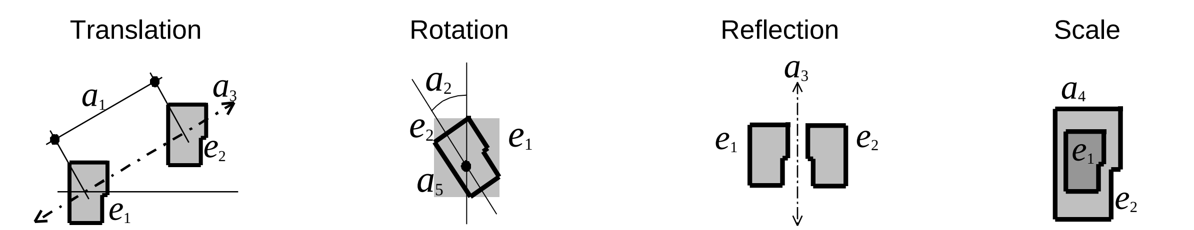
In a group of shapes, each shape can be described with respect to another shape using spatial relationships, especially isometric transformation and topological relationships. The initial or primary shape can be represented as a referent, and a relationship is represented as a predicate in a propositional shape description with arguments.

$$S = R\,\{E, A\} \tag{4}$$

Where A: arguments for relationship
E: referent shape
R: relationship between shapes
S: shape

Isometric transformation relationships: Isometric transformations are closed transformations that transform one shape into another shape without losing any properties. Isometric transformation relationships are the most fundamental spatial relationships upon which all shape representations, such as topology, shape semantics and patterns, can be founded. These are relationships between congruent shapes. There are four kinds of isometric transformations: translation, reflection, rotation and scaling. Examples and representations are given in Figure 5. Even though these relationships are well known, they are presented here for consistency and completeness.

In the translation relationship denoted by τ_1, the shape element e_2 can be described with respect to the shape element e_1 using arguments a_1 and a_3 where a_1 is the translation distance along the axis a_3. In the rotation relationship denoted by τ_2, a shape element e_2 can be represented with respect to the shape element e_1 with rotation angle a_2 and rotation centre a_5. The reflected shape element e_2 can be described from the shape e_1 with reflection relationship τ_3 and reflection axis a_3. The scale transformation represented by τ_4 changes the size of e_1 by the scale factor a_4.



$e_2 = \tau_1\{e_1, (a_1, a_3)\}$
τ_1 = translation
a_1 = distance

$e_2 = \tau_2\,\{e_1, (a_2, a_5)\}$
τ_2 = rotation
a_2 = angle
a_5 = rotation centre

$e_2 = \tau_3\,\{e_1, a_3\}$
τ_3 = reflection
a_3 = axis

$e_2 = \tau_4\,\{e_1, a_4\}$
τ_4 = scale
a_4 = scale

Figure 5. Basic isometric transformation relationships and their representations.

Using these isometric transformation relationships, more complex spatial relationships can be described. In Figure 6(a), congruent shapes are located along the axis a_3 with the same distance a_1. The axis can be a straight or a curved line. In rotation relationships, shapes are rotated around the rotation centre a_5 by the rotation angle a_2, Figure 6(b).

Composite transformations as well as single transformations of one shape into congruent shapes (or similar shape) are possible. For example, a shape can be translated along the axis a_3 with the distance a_1, at the same time it is scaled by the scale factor a_4, Figure 6(c). Two isometric transformations specify a relationship between two shapes e_1 and e_2. Three isometric transformations can specify the relationship. The shape e_1 in Figure 6(d) can be translated along the axis a_3 with the distance a_1, scaled by the scale factor a_4 and simultaneously rotated with rotation degree a_2 along the centre a_5 to make up a shape e_2.

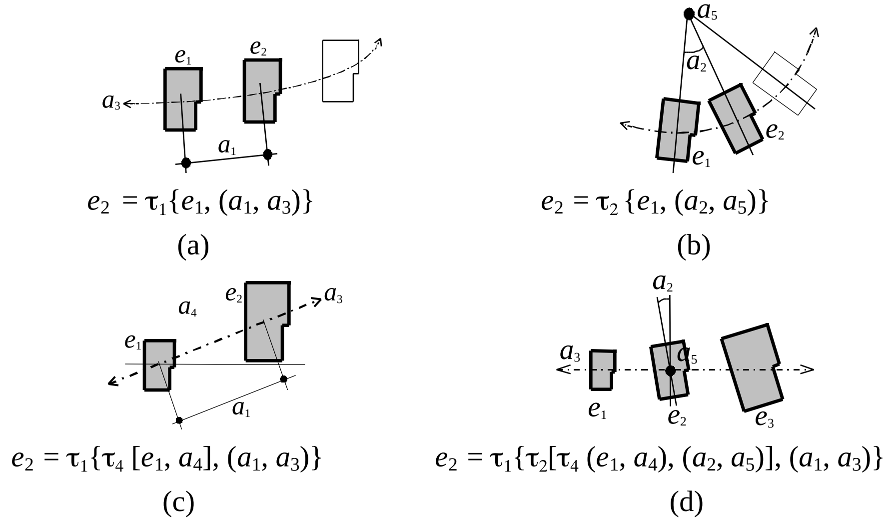


Figure 6. Single and composite transformations and their representations.

Topological relationships: Relationships between shapes are represented in terms of propositional relationships as well as isometric transformation relationships in Euclidean space. The topological relationships specify the relative positions of shapes, such as front, back, top, bottom, left, right, and so on. For example, “a triangle is left of the square,” and “a triangle is in front of the square,” where "left" and "front" are topological predicates and “square” is a referent. Some relationships are variously interpreted by different propositions such as observers, objects and environments, and topological relationships are derived from basic shape relationship descriptions, such as lower topological relationships and mathematical descriptions of spatial relationships (Gero, 1984; Olson and Bialystok, 1983). Examples of topological relationships and their representations are shown in Figure 7.

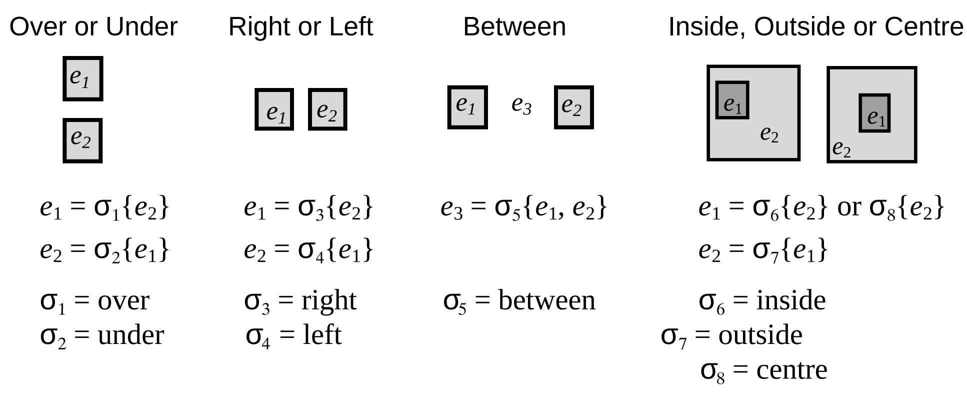


Figure 7. Topological relationships and their representations.

3.3.3 Group shape representation

A group of shapes or patterns that are congruent or similar may be arranged in a pattern, and a shape or pattern can be explained with respect to another shape or pattern recursively, then this group shape can be described using an isometric transformation relationship representation and a nesting operator ($\bigcup_{i=1}^{n}$). The nesting operator applies a transformation factor to elements recursively until all shapes or patterns in a pattern are described by another shape or pattern. It provides a description of how the given pattern is constructed from the primitives. The general representation of the relationship between two shapes or patterns is $e_i = \{e_{i-1}, a_k\}$, and a shape pattern is an arrangement of a congruent shape group or pattern group $\{e_1, e_2, \ldots, e_n\}$. Thus the pattern in this shape group can be described with a nesting operator as:

$$S = {}_{i=1}^{n}\tau_k\{e_i, a_k\} \qquad (5)$$

The nesting operator denotes n recursive applications of isometric transformation τ_k to shape elements e_i with transformation arguments a_k.

Translation pattern: In a translation pattern, a set of congruent shapes is located on the axis a_3 with distance a_1 and this pattern is represented as ${}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$, Figures 8(a) and (b).

Furthermore, a set of congruent shape patterns is grouped together and specifies a high-level translation pattern, Figure 8(c). It is represented as ${}_{j=1}^{n}\tau_1\{s_j, (a_1, a_3)\}$ where $s_j = {}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$, or ${}_{j=1}^{n}\tau_1\{ {}_{i=1}^{m}\tau_1[e_{i,j}, (a_1, a_3)], (a_1, a_3)\}$

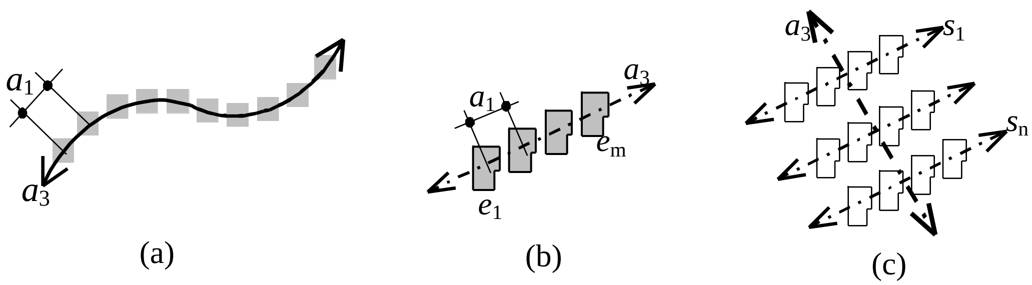


Figure 8. Translation relationship and translation pattern.

Rotation pattern: In a rotation pattern, a set of congruent shapes is rotated through a rotation centre a_5 with rotation angle a_2 and this pattern is represented as ${}_{i=1}^{m}\tau_2\{e_i, (a_2, a_5)\}$, Figures 9(a) and (b). In addition, a set of these congruent patterns is grouped together and specifies a high-level rotation pattern, Figure 9(c). It is represented as ${}_{j=1}^{n}\tau_2\{ {}_{i=1}^{m}\tau_2[e_{i,j}, (a_2, a_5)], (a_2, a_5)\}$.

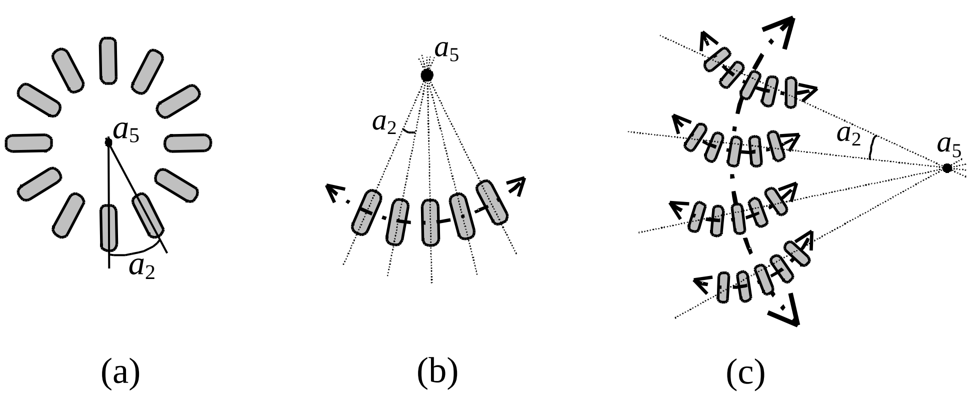


Figure 9. Rotation relationship and rotation pattern.

Reflection pattern: In a reflection pattern, two congruent reflection patterns are reflected by the reflection axis a_3, and this pattern is represented as ${}_{j=1}^{2}\tau_3\{ {}_{i=1}^{2}\tau_3(e_{i,j},\ a_3),\ a_3\}$, Figures 10(a) and (b).

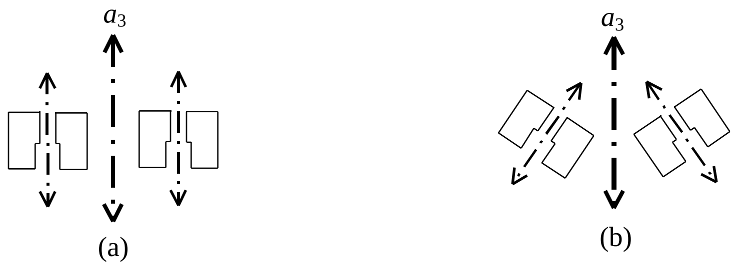


Figure 10. Reflection relationship and reflection pattern.

Scaling pattern: In a scaling pattern, a set of congruent shapes is scaled with a scale factor a_4 and this pattern is represented as ${}_{i=1}^{m}\tau_4(e_i,\ a_4)$, Figure 11(a). In addition, a set of these congruent scaled shapes is arranged on the axis a_3 with distance a_1, Figure 11(b). It is represented as ${}_{i=1}^{m}\tau_1\{\tau_4[e_{i,j},\ a_4],\ (a_1,\ a_3)\}$. Furthermore these patterns are scaled with scale factor a_4 and translated along the axis a_3 with distance a_1, Figure 11(c). It is represented as ${}_{j=1}^{n}\tau_1\{\tau_4[\ {}_{i=1}^{m}\tau_1\{\tau_4(e_{i,j},\ a_4),\ (a_1,\ a_3)\},\ a_4],\ (a_1,\ a_3)\}$.

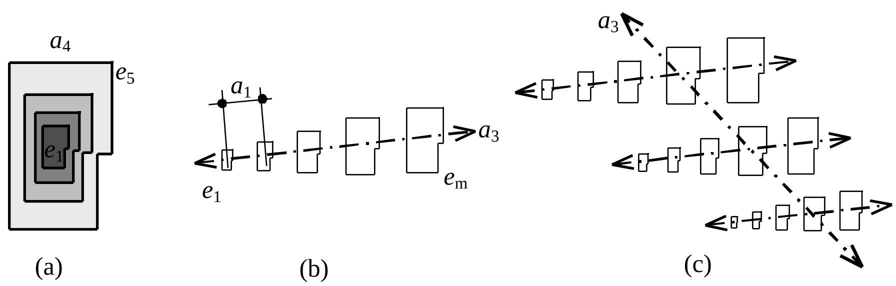
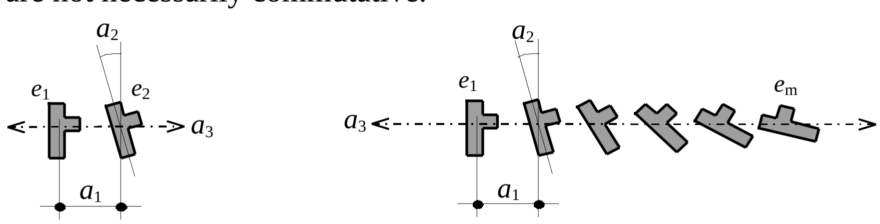


Figure 11. Scaling relationships and scaling pattern.

In the descriptions of the basic isometric transformations in Figure 12, predicates encapsulated by nesting operators are commutative; that is, if τ_k and τ_j are any two basic isometric transformation relationships, then $\tau_k * \tau_j = \tau_j * \tau_k$. In Figure 12, the transformation factors τ_1 and τ_2 are applied to a shape e_{i-1} to produce a shape e_i. the order of application does not affect the results. Thus, shape descriptions S_a and S_b produced by recursive applications of τ_1 and τ_2 are identical. However, composite isometric transformation relationships such as in Figure 13 are not necessarily commutative.



$e_2 = \tau_2\{\tau_1\,[e_1,\ (a_1,\ a_3)],\ (a_2,\ a_5)\}$ $S_a = {}_{i=1}^{m}\tau_2\{\tau_1\,[e_i,\ (a_1,\ a_3)],\ (a_2,\ a_5)\}$

$e_2 = \tau_1\{\tau_2\,[e_1,\ (a_2,\ a_5)],\ (a_1,\ a_3)\}$ $S_b = {}_{i=1}^{m}\tau_1\{\tau_2\ [e_i,\ (a_2,\ a_5)],\ (a_1,\ a_3)\}$

Figure 12. Predicates between nesting operators are commutative.

In addition, predicates encapsulated by nesting operators are not commutative with predicates out of nesting operators; that is, if τ_k and τ_j are any two isometric transformation relationships, then $\tau_k \; {}^{n}_{i=1}\tau_j \neq \tau_j \; {}^{n}_{i=1}\tau_k$. Different orders of predicates over nesting operators produce different results. In Figure 13(a), a transformation factor τ_1 is applied to a shape recursively and produces a linear transformation of that shape. Then a rotation transformation is applied to a set of linear transformations to generate a shape group in Figure 13(b). Switching of transformation factors generates different shapes as shown in Figures 13(c) and (d).

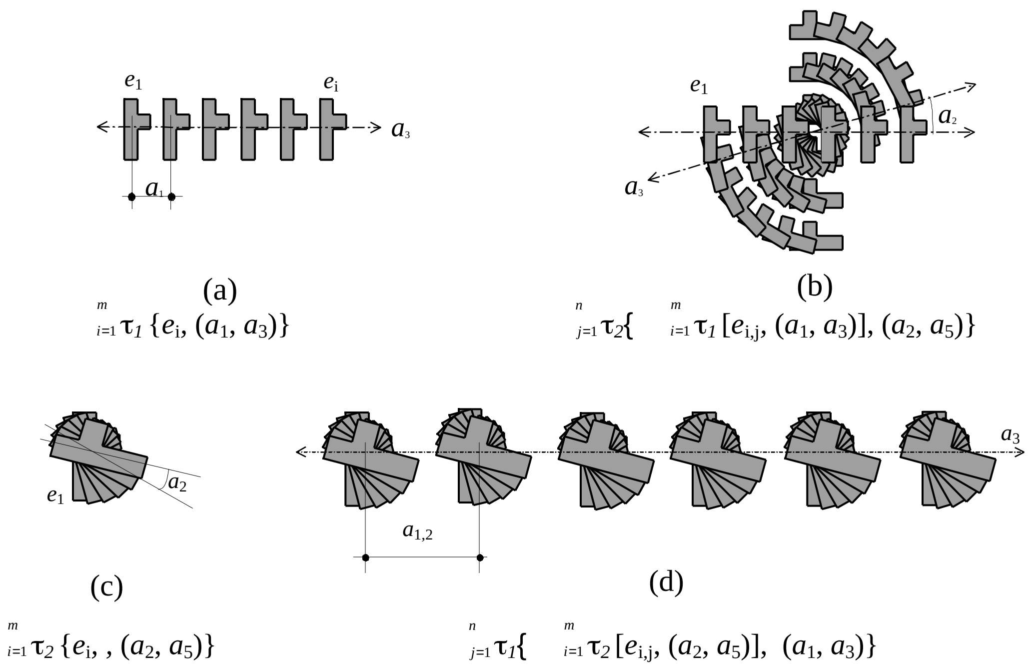


Figure 13. Switching of transformation factors produces different shapes.

The nesting operator (${}^{n}_{i=1}$) separates super-nodes and sub-nodes in a hierarchical tree structure and the position of the nesting operator depends on the level of spatial relationships. Predicates after the nesting operator describe spatial relationships of a set of next low-level subnodes. The nesting operator distinguishes each branch in the hierarchical tree structure as will be seen later in Section 3.3.5.

3.3.4 Relationships of relationships

Spatial relationships can be specified from a set of low-level relationships as well as from physical shapes. A set of congruent or similar relationships or semantics can be grouped together and specify high-level relationships recursively. Low-level relationships specified from a set of congruent shapes are clustered in terms of their congruency, and identify high-level relationships. Furthermore, a set of similar relationships constructed either from different shape groups or from different relationship groups can specify high-level relationships.

Primitives or group shapes may have their own internal relationships or semantics that can be identified with labelled interpretations, such as self-reflection, axiality, centrality, spirality, as shown in Figure 14.

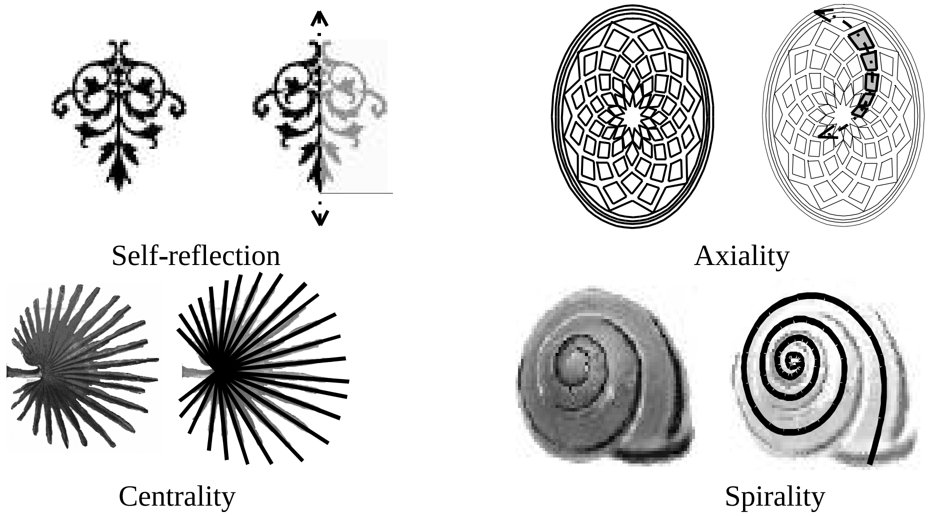


Figure 14. Internal relationships or semantics of shape elements.

A set of similar internal relationships or semantics that can be identified by interpretation rather than physical properties specifies high-level relationships. In Figure 15, individual shapes are not congruent, but the patterns interpreted as wholes have the same semantics, called axiality. Each shape has its own internal axis. If a set of these axes is arranged in a certain relationship, they can specify a high-level relationship. Five axes in Figure 15 are parallel and located with the same interval. They are in a translation relationship.

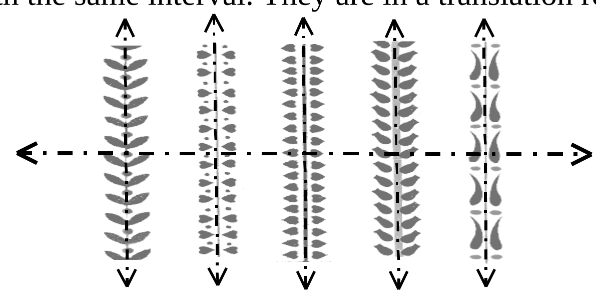


Figure 15. Relationship identified from internal relationships or semantics.

A spatial relationship can be described not only from congruent shapes but also from congruent relationships. If relationships identified from sets of congruent shapes are congruent and arranged with a defined relationship, they can identify high-level relationships. In Figure 16, sets of congruent shape groups specify two gradation relationships. These two gradation relationships are congruent and reflected, thus they can be described as a reflection of two gradation relationships.

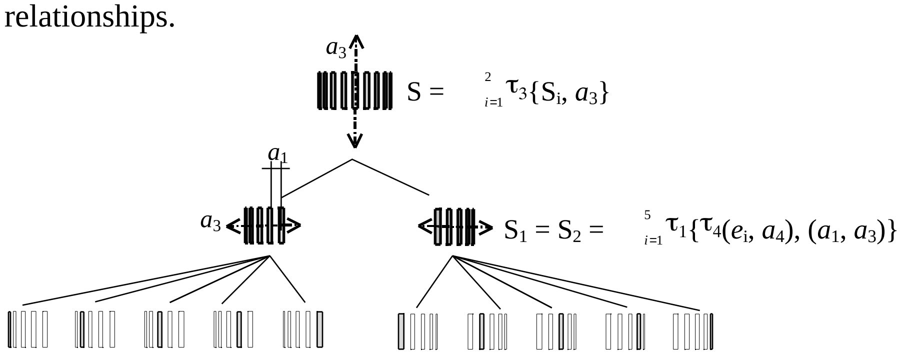
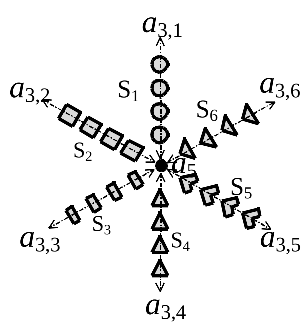


Figure 16. Relationship identified from congruent spatial relationships.

Different shape groups may share the same relationships. In Figure 17, different sets of similar shapes are grouped together under the same linear transformation relationship. These relationships are congruent even though the low-level shape elements are different.



$S_1 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,1})\}$ $e_i =$

$S_2 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,2})\}$ $e_i =$

$S_3 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,3})\}$ $e_i =$

$S_4 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,4})\}$ $e_i =$

$S_5 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,5})\}$ $e_i =$

$S_6 = {}^{4}_{i=1}\tau_1\{e_i, (a_1, a_{3,6})\}$ $e_i =$

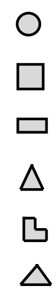
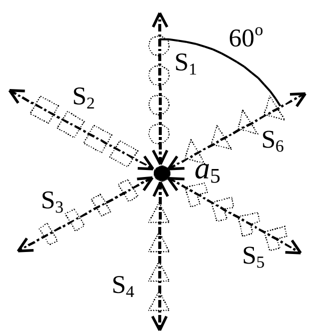


Figure 17. Six different sets of shapes are grouped together independently and specify the same translation relationship.

From these six different shape groups, only relationships are considered to construct a high-level shape relationship. Properties of shapes are disregarded, and shapes are considered as variables. Six congruent relationships in a pattern identify the 60° rotation relationship. Thus relationships are constructed not only from similar shapes but also from similar relationships as shown in Figure 18.



$$S = {}^{6}_{i=1}\tau_2\{S_i, (60^{\circ}, a_5)\}$$

Figure 18. Relationships from sets of shape groups are congruent, and are grouped together to identify a high-level relationship.

3.3.5 Hierarchical representation of shape patterns

Spatial relationship elements as well as physical shape elements specify high-level relationships recursively, and then a set of relationships is composed in the form of a hierarchical tree structure. Construction of a hierarchical tree structure from primitive shapes depends on the similarities between relationships as well as shape congruency. Spatial relationship similarities are identified by predicates and arguments in shape descriptions. Sets of similar relationships in a pattern are grouped together and specify high-level relationships recursively. Suppose a shape that can be decomposed into many subshapes as shown in Figure 19. It is composed of four group shapes that specify a rotation relationship. Four group shapes are constructed from reflections of two shape groups. Furthermore, reflected shape groups are composed of 90° rotations of four elementary shapes.

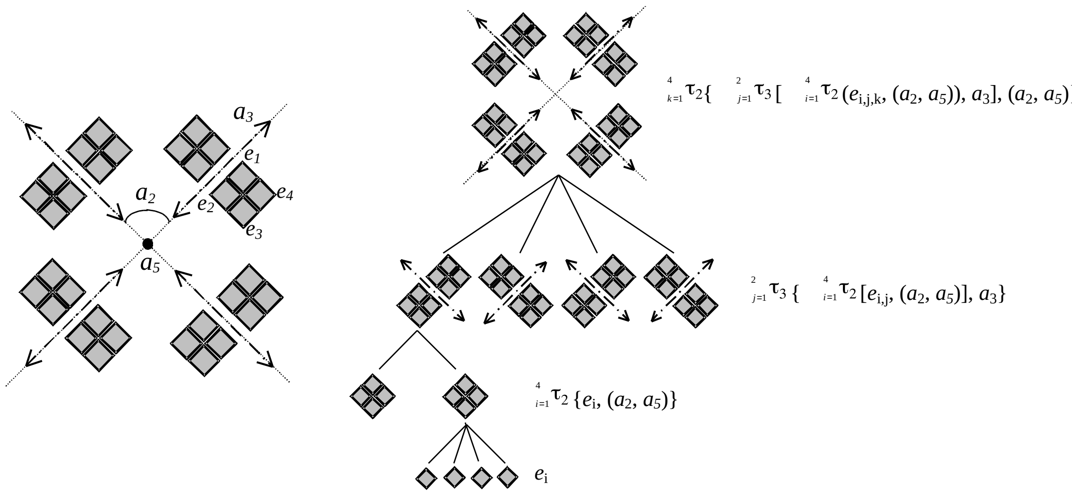


Figure 19. Hierarchical tree structure of a shape pattern representation.

4 Similarity in shapes and shape patterns

According to Vosniadou and Ortony (1989), there are two kinds of similarity, surface similarity and deep similarity. Surface similarity is cognitively primitive and well defined, and can be used as a constructor to explain other psychological functions such as categorisation (Rips, 1989). Deep similarity is a similarity with respect to more central, core properties of concepts. Gentner (1989) makes a distinction between object attributes and relations. Surface similarity is based on shared object attributes, and structural similarity is similarity at the level of relational structure. Based on these two similarities, more kinds of similarities may be introduced, such as analogy, mere appearance similarity, literal similarity, metaphor, and so on.

4.1 Kinds of Similarity

Similarities in shapes are identified by attributes, physical structure (Gero and Jun, 1995), continuous transformation (March and Steadman, 1971; Steadman, 1983; Mitchell, 1990), or organising structure (Falkenhainer *et al*, 1989/90). Shapes are recognised and categorised in terms of their attributes, such as colour, line type, thickness, and so on. Shapes that have the same physical structure in terms of topology and geometry are regarded as similar shapes: congruent shapes. They are transformed in various ways, for example, stretch, shear, perspective or rubber sheet. Group shapes that are composed of different subshapes but have the same compositional relationships are called analog shapes.

Mere appearance shapes share the same attributes. An attribute refers to any single component or property of an object, such as colour, material, line style, etc. For example, if the same colour shapes are grouped together perceptually, their similarity is identified by the same colour and the colour characterises the group shape.

Mere appearance shapes can be identified from shape representation that describes properties of shapes using predicate calculus (Coyne *et al*, 1990). The yellow colour of a window can be described as Colour (a window, yellow) and the yellow colour of a door can be described as Colour (a door, yellow). From these two colour attribute descriptions, their similarity in terms of mere appearance can be identified because they share the same value (yellow) for the colour attribute.

Congruent shapes have the same structure of elements in terms of topology and geometry. If two shapes have the same number of infinite maximal lines, number of intersections, geometrical properties of infinite maximal lines and dimensional constraints on segments on each infinite maximal line, then these two shapes are congruent (Gero and Jun, 1995).). In Figure 20, all group shapes are composed of subshapes: circles and diamond shapes. Corresponding subshapes in each group shape have the same physical structure, thus corresponding subshapes are regarded as congruent shapes. Furthermore subshapes in each group shape are arranged in the same manner such that group shapes are also congruent. All shapes in Figure 20 have the same physical structure, but they are differentiated by their attributes, locations, directions and sizes.

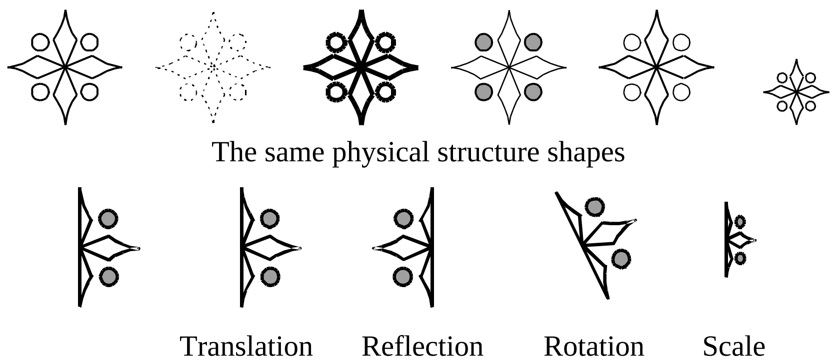


Figure 20. Congruent shapes.

Continuous transformation shapes have the same structure, but different in size or dimensional constraints. Continuous transformation shapes are distorted but always preserve the metric properties of connectedness. In the dimensional sense, they are not perfectly satisfactory or perfectly regular deformations, but nevertheless, they are symmetrical to the eye, and approach to an isogonal system under certain conditions. The biologist D'Arcy Thompson (1952) showed that distortions of dimensional property with constant structural properties produce cognitively similar shape, Figure 21.

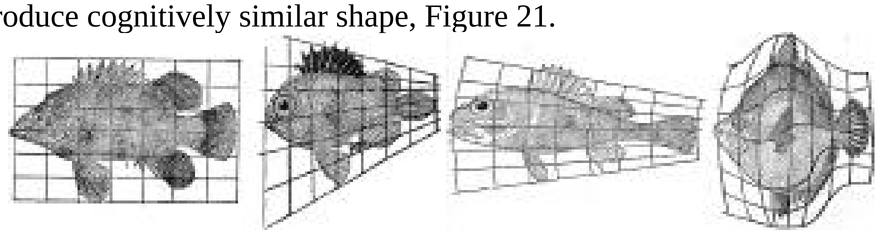
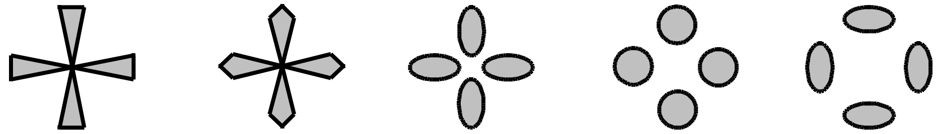


Figure 21. Similarity in continuous transformation (after Thompson, 1952).

Analog shapes have the same organising structure, but which may have different subshapes. In Figure 22, all shapes have different physical structures, sizes, locations, etc., but they have the same spatial relationship, which is a 90^{o} rotation of four shapes. The shape in Figure 22(a) is constructed from four triangles, the shape in Figure 22(b) is composed of four quadrilaterals, the shape in Figure 22(c) is made of four ovals, and so on. All the shapes share the same organising structure, thus they are analog shapes.

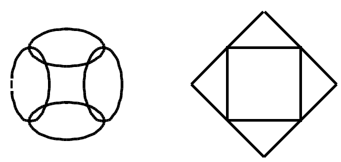


(a) (b) (c) (d) (f)

Figure 22. Analog shapes.

4.2 Similarity in shape patterns

Representations of shapes and their relationships were presented in Section 3. Shapes are decomposed into subshapes, and represented with shape elements e_i and spatial relationships τ_i with arguments a_i. Structural similarity between shapes can be decided from these shape descriptions. Properties of shape elements e_i are disregarded in structural matching, and are generalised as element variables. Predicates and arguments are considered to determine similarity. Consider the two shapes, S_a and S_b in Figure 23, to determine their similarity. In terms of surface similarity, there are no common properties, shape S_a is composed of curves or ovals, while shape S_b is composed of straight lines, triangles or squares.



S_a S_b

Figure 23. Two shapes used in similarity example.

But if the shapes are decomposed and described using sub-shapes and their relationships, then their similarity can be identified. The shape S_a in Figure 23 is composed of four ovals that have a 90^o rotation relationship, and can be described as $S_a = {}_{i=1}^{4}\tau_2\{Oval_i, (90^o, a_5)\}$. The shape S_b in Figure 23 is composed of four triangles that have a 90^o rotation relationship, and can be described as $S_b = {}_{i=1}^{4}\tau_2\{Triangle_i, (90^o, a_5)\}$. Comparing these two shape descriptions, it is evident that they share the same predicate (τ_2) and arguments (90^o, a_5), thus they can be regarded as similar shape patterns in terms of the relationship, even though the physical properties in the element shapes are different, Figure 24.

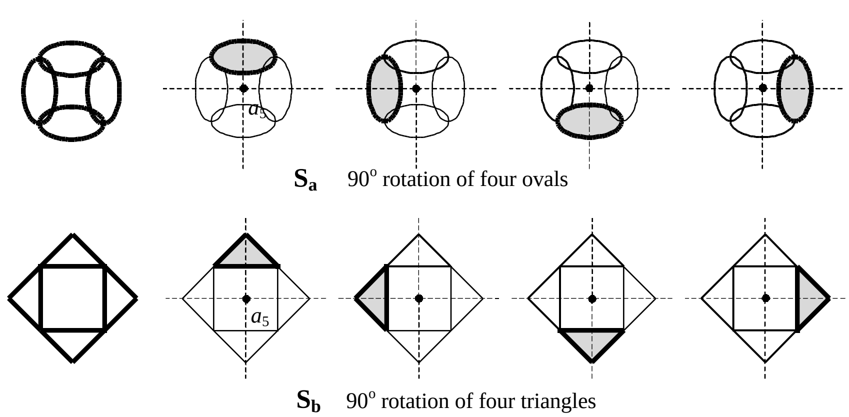
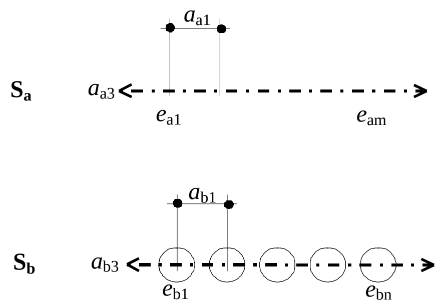


Figure 24. 90^o rotation relationship of shape S_a and S_b.

Based on structural similarity, similarity of isometric transformation relationships can be identified from shape pattern descriptions. The translation pattern is represented using a translation distance and a translation axis. The congruent translation pattern exists between two shape patterns if and only if the two shape pattern descriptions have the same predicates, types of axes, distances between sub-elements and the same numbers of sub-elements, Figure 25.



$S_a = {}_{i=1}^{m} \tau_a\{e_{ai}, (a_{a1}, a_{a3})\}$

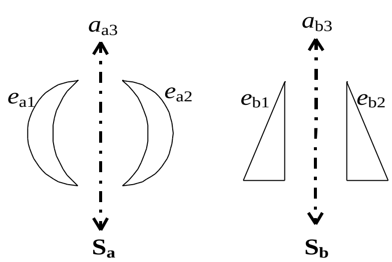
$S_b = {}_{i=1}^{n} \tau_b\{e_{bi}, (a_{b1}, a_{b3})\}$

if $(m = n)$
$(\tau_a = \tau_b = \text{Translation})$
$(a_{a1} = a_{b1})$
$(a_{a3} = a_{b3})$

Then congruent translation pattern **(S_a, S_b)**

Figure 25. Congruent translation patterns.

The reflection pattern is represented using a reflection axis. A congruent reflection pattern exists between two shape patterns if and only if the two shape pattern descriptions have the same predicates, the same numbers of sub-elements and the same types of axes, Figure 26.



$S_a = {}_{i=1}^{m} \tau_a\{e_{ai}, a_{a3}\}$

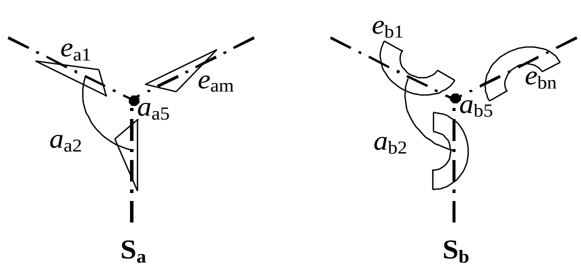
$S_b = {}_{i=1}^{n} \tau_b\{e_{bi}, a_{b3}\}$

if $(m = n)$
$(\tau_a = \tau_b = \text{Reflection})$
$(a_{a3} = a_{b3})$

Then congruent reflection pattern **(S_a, S_b)**

Figure 26. Congruent reflection patterns.

The rotation pattern is represented using a rotation centre and a rotation angle. A congruent rotation pattern exists between two shape patterns if and only if the two shape pattern descriptions have the same numbers of sub-elements and the same rotation angles, Figure 27.



$S_a = {}_{i=1}^{m} \tau_a\{e_i, (a_{a2}, a_{a5})\}$

$S_b = {}_{i=1}^{n} \tau_b\{e_i, (a_{b2}, a_{b5})\}$

if $(m = n)$
$(\tau_a = \tau_b = \text{Rotation})$
$(a_{a2} = a_{b2})$

Then congruent rotation pattern **(S_a, S_b)**

Figure 27. Congruent rotation patterns.

Similarity between complex shape pattern descriptions represented in the form of a hierarchical tree structure can be identified by comparing the highest predicates and arguments, then moving down to lower predicates and arguments. If two shape pattern descriptions have the same predicates and arguments, they can be considered as similar shape patterns in terms of relationships, even though they may have different lower predicates and arguments. Their lower shapes and patterns can be generalised.

Shapes in Figure 28 are composed of different subshapes and low-level relationships, even though they share the same high-level patterns. Circles are arranged in a pattern, Figure 28(a) top, and L-shapes are arranged in a certain way, Figure 28(b) top. The three circles in Figure 28(a) specify a scale pattern in which sizes of circles are increased by the scale factor a_4 and this pattern is represented as $_{i=1}^{3}\tau_4\{e_i, a_4\}$. These four congruent scale patterns are rotated through the rotation centre a_5 by the rotation angle a_2 and the rotation pattern is represented as $_{j=1}^{4}\tau_2\{x_j, (a_2, a_5)\}$ where x_j are low-level elements ($_{i=1}^{3}\tau_4\{e_i, a_4\}$). Then three congruent rotation patterns are translated along the axis a_3 with the distance a_1 and the result is represented as $_{k=1}^{3}\tau_1\{x_k, (a_1, a_3)\}$. This shape pattern is in the form of a hierarchical tree structure. In addition, the two L-shapes in Figure 28(b) are reflected around the reflection axis a_3 and this reflection pattern is represented as $_{i=1}^{2}\tau_3\{e_i, a_3\}$. These four reflected patterns are congruent and rotated through the rotation centre a_5 by the rotation angle a_2, and the rotation pattern is represented as $_{j=1}^{4}\tau_2\{x_j, (a_2, a_5)\}$. The three congruent rotated patterns are arranged on the axis a_3 with the translation distance a_1 ($_{k=1}^{3}\tau_1\{x_k, (a_1, a_3)\}$).

If the high-level patterns are compared, a congruent shape pattern can be identified even though the low-level shapes and patterns are different. The highest pattern in Figure 28(a) is a translation of three shape patterns. Also, the pattern in Figure 28(b) is a translation of three shape patterns. Thus, these two shape groups share the same highest shape pattern, that is, a translation of three shape pattern elements which is represented as $_{k=1}^{3}\tau_1\{x_k, (a_1, a_3)\}$. The three translated shape patterns in these two congruent shape patterns are composed of four low-level shape patterns in a specified a rotation pattern. They are congruent again and are represented as $_{j=1}^{4}\tau_2\{x_j, (a_2, a_5)\}$, but their lowest shape patterns and shapes are different. The lowest shape patterns in Figure 28(a) are scale patterns of three circles ($_{i=1}^{3}\tau_4\{e_i, a_4\}$) and the lowest shape patterns in Figure 28(b) are reflection patterns of L-shapes ($_{i=1}^{2}\tau_3\{e_i, a_3\}$).

Even though each shape pattern has different low-level patterns and shapes, they can be considered as similar shape patterns in terms of their high-level pattern congruency. Some shape patterns may share the same relationships and arguments from the highest to the lowest in the hierarchical tree structure. The only differences may be at the level of the lowest physical shapes. Their predicates and arguments in shape descriptions are the same, but the lowest shapes are different. These shape patterns are considered to be complete congruent shape patterns.

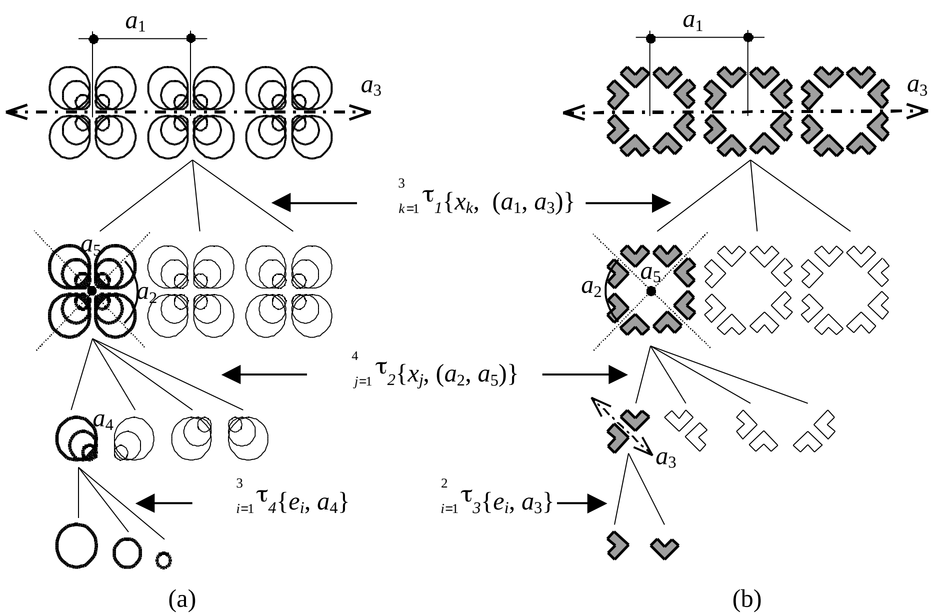
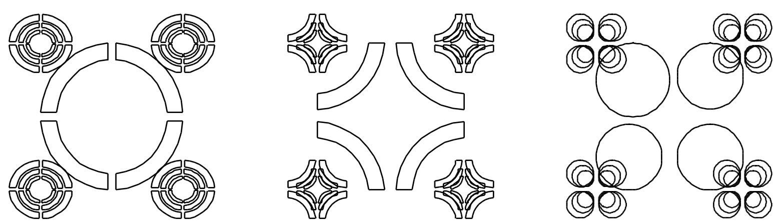


Figure 28. Similarity in high-level patterns between complex shape patterns.

Shape patterns which share the same relationships and arguments from the highest to the lowest in a hierarchical tree structure with differences at the lowest physical shape elements are congruent shape relationships. The shape patterns in Figure 29 are composed of different subshapes. But they are organised in the same pattern. Their predicates and arguments in their shape descriptions are the same, and only the lowest shape elements are different. These shape patterns are considered as congruent shape pattern relationships.



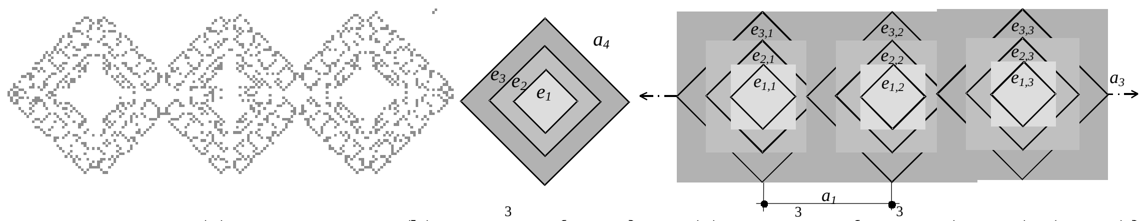
$${}_{k=1}^{4}\tau_2\{\ {}_{j=1}^{3}\tau_4\{\ {}_{i=1}^{4}\tau_2\,[e_{i,j,k}, (a_2, a_5)], a_4\}, (a_2, a_5)\}\ \text{and}\ {}_{i=1}^{4}\tau_2\{e_i, (a_2, a_5)\}$$

Figure 29.Congruent shape pattern relationships in complex shape patterns.

5 Shape patterns and representations in architectural drawings

We can use the symbolic representations developed in the previous sections to describe, in a potentially computable form, complex architectural drawings that are otherwise too difficult to represent as a simple conjunction of elements.

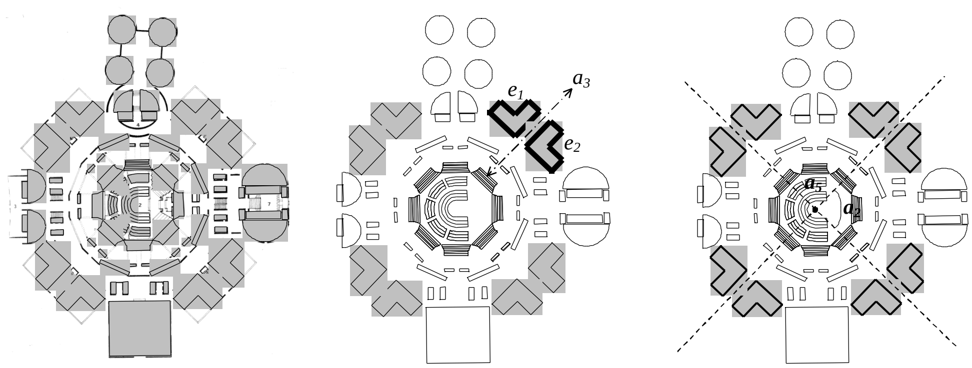
Figure 30(a) shows an architectural plan of Erdman Hall Dormitory designed by Louis I. Kahn. Figures 30(b) and (c) show the shape elements, and one possible symbolic representation of those elements and their relationships, which go to make up this plan. Thus, the plan can be represented through a combination of scale and translation transformations of constant elementary shapes and relationships. Three rhombuses in Figure 30(b) have a scale transformation relationship that is specified by a shape element e_i and a scale factor a_4. The scale of an initial shape e_1 is increased in terms of the scale factor a_4 and this pattern is represented as $^{3}_{i=1}\tau_4\{e_i,\ a_4\}$. Furthermore, these three patterns identified by scale transformation relationships are arranged along a linear axis a_3 with the same distance a_1, thus produce a translation relationship τ_1. This complex shape pattern is represented as $^{3}_{j=1}\tau_1\{\ ^{3}_{i=1}\tau_4(e_{i,j}, a_4), (a_1, a_3)\}$.



(a) (b) = $^{3}_{i=1}\tau_4\{e_i, a_4\}$ (c) = $^{3}_{j=1}\tau_1\{\ ^{3}_{i=1}\tau_4(e_{i,j}, a_4), (a_1, a_3)\}$

Figure 30. Shape pattern and representations: (a) Erdman Hall Dormitory by Louis I. Kahn, (b) and (c) representations of the drawing in (a).

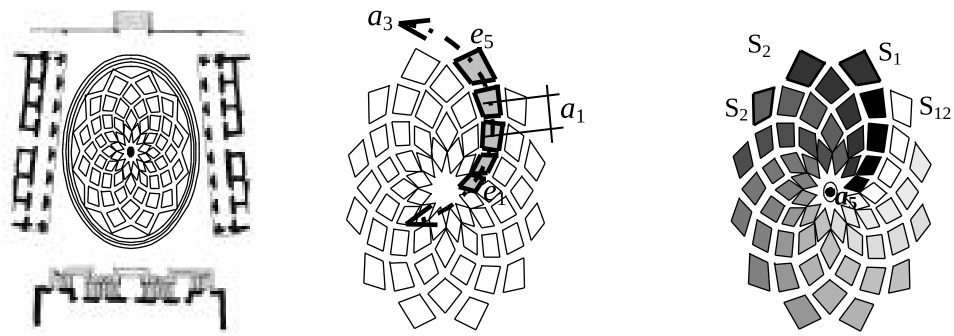
Figure 31(a) shows a floor plan of the National Assembly building in Dacca also designed by Louis I. Kahn. Figures 31(b) and 31(c) show the shape elements, and one possible symbolic representation of those elements and their relationships which go to make up a part of the floor plan. Thus, one representation of this floor plan is through a combination of reflection and rotation transformations of the elementary shapes and relationships. Two L-shapes e_1 and e_2 in Figure 31(b) are reflected by reflection axis a_3 and this pattern is represented as $^{2}_{i=1}\tau_3(e_i, a_3)$. In addition, four reflected shape patterns are congruent and arranged in another higher pattern that is a rotation transformation. Four reflected shape patterns are rotated through the angle a_2 around the centre a_5, represented as $^{4}_{j=1}\tau_2\{\ ^{2}_{i=1}\tau_3(e_{i,j}, a_3), (a_2, a_5)\}$.



(a) (b) $^{2}_{i=1}\tau_3(e_i, a_3)$ (c) $^{4}_{j=1}\tau_2\{\ ^{2}_{i=1}\tau_3(e_{i,j}, a_3), (a_2, a_5)\}$

Figure 31. Shape patterns and representations: (a) the National Assembly building, Dacca (Louis I. Kahn), (b) reflection of two L-shapes, (c) rotation of four reflection shape patterns.

Figure 32(a) shows a plaza plan of the Campidoglio designed by Michelangelo. Figures 32(b) and (c) show the shape elements and one possible symbolic representation of those elements and patterns which go to make up this plaza plan. Thus, one pattern representation of this plaza plan is through a combination of rotational, translational and scaling transformations of the elementary shapes. Five quadrilaterals in Figure 32(b) are arranged in a complex transformation. Three transformation factors are applied to produce this shape pattern. Size of shapes increases from e_1 to e_5 by a scale factor a_4, simultaneously, shapes are self-rotated through the angle a_2 and translated along the curved axis a_3 with distance a_1. Furthermore, twelve composite patterns are congruent and specify a higher pattern that is a rotation transformation. Twelve shape patterns are rotated through the angle a_2 around the centre a_5 and represented as ${}_{j=1}^{12}\tau_2\ \{S_j, (a_2, a_5)\}$.



$S_j = {}_{i=1}^{5}\tau_1\ \{\tau_2\ [\tau_4\ (e_i, a_4), (a_2, a_5)], (a_1, a_3)\}$ Campidoglio = ${}_{j=1}^{12}\tau_2\ \{S_j, (a_2, a_5)\}$

Figure 32. Shape pattern and representations: (a) the Campidoglio (Michelangelo), (b) and (c) representations of the drawing in (a).

6 Shape pattern schema representation

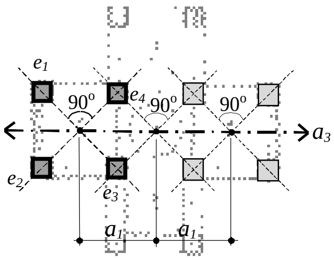
A shape pattern schema is an organised body of knowledge about spatial relationships between shapes that describes the patterns, syntactic structure, and the characteristics of shape patterns. In perceptual categories of shapes, objects are perceived not only by their properties, such as colour and material, but also by their organising structures specified by spatial relationships. Perceptual categories exist by virtue of similar structural descriptions. According to Rumelhart (1980), a schema is "a higher order relational structure for representing the generic concepts upon which all information processings depend". It is a network of inter-relationships that represents essential characteristics of things or concepts rather than a list of features. The network may be in a hierarchical tree structure with nodes and paths. It is generalised from multiple repetitions (Piaget, 1952) and describes a prototype (or a generic concept) for a group of things or situations (Minsky, 1975).

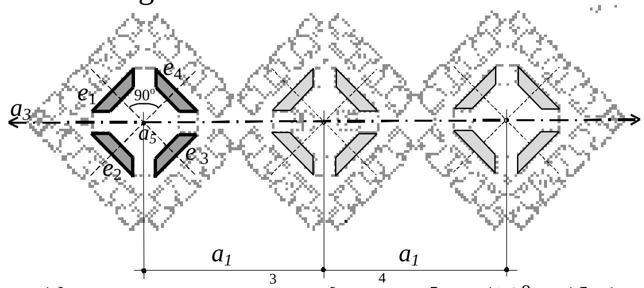
The shape pattern schema is represented by a set of sub-elements and their relationships. It is generalised from multiple shape pattern representations or a class of shape pattern descriptions. It can be in the form of a hierarchical tree structure, and the low-level nodes in a hierarchical tree structure can be turned into variables and embedded in each other.

6.1 Variability in shape pattern schemas

A shape pattern schema is not a representation of a specific shape pattern, rather it represents spatial characteristics for a set of shape patterns. Each shape pattern is a specific instantiation of the shape pattern schema. From the notion of schema as a network of elements and relationships, lower level schemas as well as lower level elements and relationships can be regarded as variables that can be instantiated. In shape pattern schemas, shape elements and lower level relationship elements or schemas are considered as variables.

Consider the two shape patterns that are described as N_1 and N_2 in Figure 33. These two patterns can be generalised into a shape pattern schema S_1 that is a translation of three 90^o rotation relationships. High-level relationships, such as the translation and 90^o rotation are fixed, but low-level elements and arguments can be turned into variables.





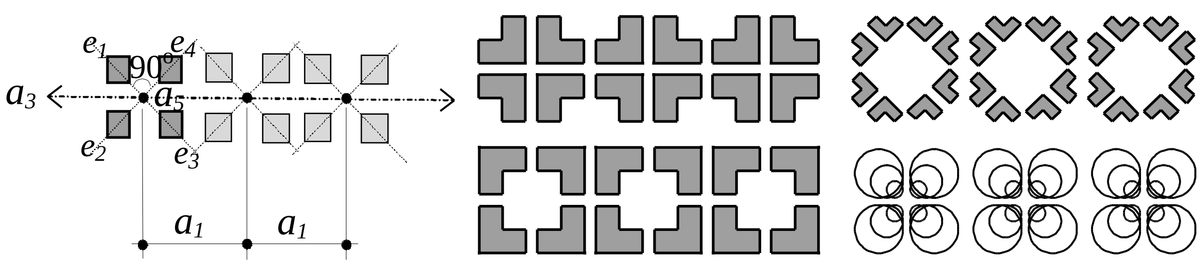
$N_1 = {}^{3}_{j=1}\tau_1 \{ {}^{4}_{i=1}\tau_2 [e_{i,j}, (90^o, a_5)], (a_1, a_3)\}$ $N_2 = {}^{3}_{j=1}\tau_1 \{ {}^{4}_{i=1}\tau_2 [e_{i,j}, (90^o, a_5)], (a_1, a_3)\}$

$S_1 = {}^{3}_{j=1}\tau_1 \{ {}^{4}_{i=1}\tau_2 [x_{ei,j}, (a_2, a_5)], (x_{a1}, a_3)\}$

Figure 33. Shape patterns and their shape pattern schemas.

Instantiation of variables in generalised shape patterns can generate many different shape patterns, as shown in Figure 34. Variables can be instantiated in two ways: they can be turned into shapes or sub-patterns. The instantiated sub-patterns can be further instantiated until physical shapes are instantiated as the lowest elements.

Drawings in Figure 34(b) show instantiation of variables into physical shapes. Drawings in Figure 34(c) show instantiations of variables into sub-patterns. All shapes in Figures 34(b) and (c) are constructed from different shape elements or sub-patterns, but they share a commonality that is a translation of four rotation patterns. In terms of the structural similarity, they can be classified into the same category.



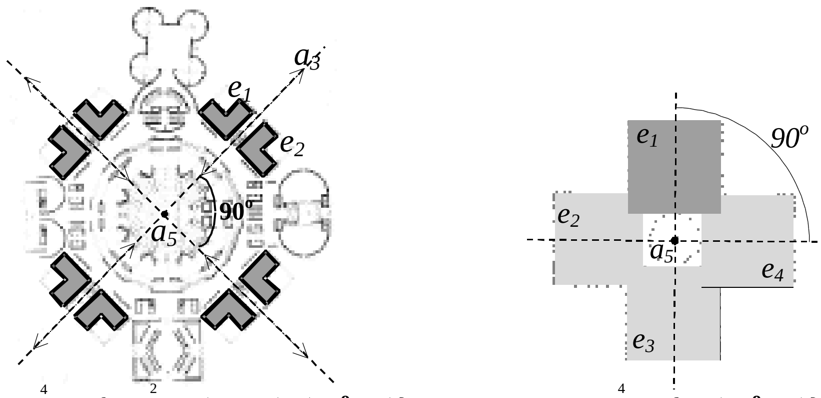
(a) (b) (c)

Figure 34. Shape pattern schema and instantiations; (a) A shape pattern schema, and (b) and (c) example instantiations of this shape pattern schema.

6.2 *Embeddedness in shape pattern schemas*

One schema may be embedded in one or many others in various ways. A schema may be a part of other schema, many schemas may share a subschema, or a schema can be embedded within itself.

Part: A subschema or a set of subschemas is embedded in a shape pattern schema if that subschema is a part of this shape pattern or schema. Drawings in Figure 35 have the same high-level shape patterns, thus shape patterns of these two shapes can be generalised into a shape pattern schema that is a 90^o rotation of four elements. It is represented as $S_2 = {}^{4}_{i=1}\tau_2\,\{x_{ei}, (90^o, a_5)\}$.



$N_3 = {}^{4}_{j=1}\tau_2\,\{\ {}^{2}_{i=1}\tau_3\,(e_{i,j}, a_3), \mathbf{(90^o, a_5)\}}$ $\qquad N_4 = {}^{4}_{i=1}\tau_2\,\mathbf{\{e_i, (90^o, a_5)\}}$

$S_2 = {}^{4}_{i=1}\tau_2\,\{x_{ei}, (90^o, a_5)\}$

Figure 35. Shape patterns and their schemas.

Suppose we have two shape pattern schemas S_1 and S_2. Shape schema S_1 is a translation of three 90^o rotation relationships in Figure 33, and shape schema S_2 is a 90^o rotation relationship of four shape elements, Figure 35. By comparing the two shape pattern schemas S_1 and S_2 from shape pattern representations, an embeddedness can be found. The shape pattern schema S_2 is a part of the shape pattern schema S_1 so that the shape pattern schema S_2 is embedded in the schema S_1.

$S_1 = {}^{3}_{j=1}\tau_1\,\{\ {}^{4}_{i=1}\tau_2\,\mathbf{[x_{ei,j}, (90^o, a_5)]}, (x_{a1}, a_3)\}$ $\qquad S_2 = {}^{4}_{i=1}\tau_2\,\mathbf{\{x_{ei}, (90^o, a_5)\}}$

$S_2 \quad S_1$

Sharing: Many shape pattern schemas may share a single shape pattern schema or a set of shape pattern schemas. If two complex shapes overlap each other, the conjunction part of two shapes is represented with a shape pattern schema or a set of shape pattern schemas. These shared shape pattern schemas are embedded in shape pattern schemas of those complex shapes.

$N_5 = {}^{4}_{k=1}\tau_2\{\ {}^{2}_{j=1}\tau_3\,[\ {}^{4}_{i=1}\tau_2\,\mathbf{(e_{i,j,k}, (90^o, a_5)), a_3}], (90^o, a_5)\}$ $\qquad N_6 = {}^{2}_{j=1}\tau_3\,\mathbf{\{}\ {}^{4}_{i=1}\tau_2\,\mathbf{[e_{i,j}, (90^o, a_5)], a_3\}}$

$S_3 = {}^{2}_{j=1}\tau_3\,\{\ {}^{4}_{i=1}\tau_2\,[x_{ei,j}, (90^o, a_5)], a_3\}$

Figure 36. Shape patterns and their schema.

For example, a shape pattern schema S_1 is constructed from the shape pattern descriptions N_1 and N_2 in Figure 33, and a shape pattern schema S_3 can be constructed from two shape pattern descriptions N_5 and N_6 in Figure 36. The first shape pattern N_5 in Figure 36 is composed of three levels of patterns that are two rotation patterns and one reflection pattern, and is represented as $N_5 = {}_{k=1}^{4}\tau_2\{ {}_{j=1}^{2}\tau_3 [{}_{i=1}^{4}\tau_2 (e_{i,j,k}, (90^o, a_5)), a_3], (90^o, a_5)\}$. The second pattern N_6 in Figure 36 is a reflection of rotation pattern, and is represented as $N_6 = {}_{j=1}^{2}\tau_3 \{ {}_{i=1}^{4}\tau_2 [e_{i,j}, (90^o, a_5)], a_3\}$. From these two shape patterns a shape pattern schema S_3 can be generalised, and is represented as $S_3 = {}_{j=1}^{2}\tau_3 \{ {}_{i=1}^{4}\tau_2 [x_{ei,j}, (90^o, a_5)], a_3\}$.

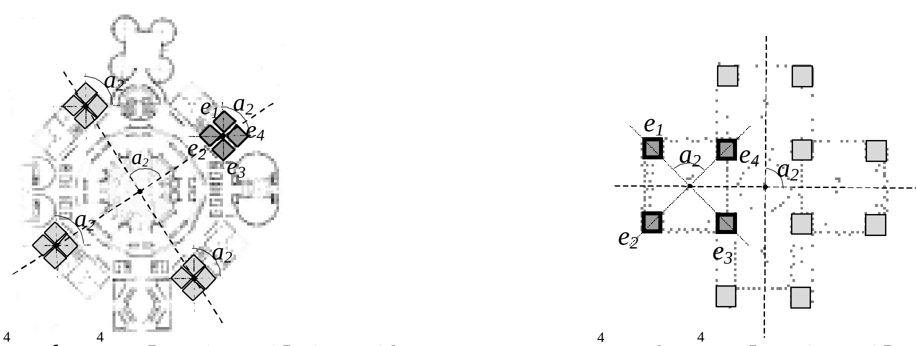
Two shape pattern schemas S_1 and S_3 share a sub-schema S_4. A part of shape schema S_1 overlaps on the shape schema S_3. The conjunction part of these two shape pattern schemas is a sub-schema S_4 that is a 90^o rotation of four shape elements. Therefore, the sub-schema S_4 is embedded in the schemas S_1 and S_3.

$S_1 = {}_{j=1}^{3}\tau_1 \{ \underline{{}_{i=1}^{4}\tau_2 \mathbf{[x_{ei,j}, (90^o, a_5)]}}, (x_{a1}, a_3)\}$ $\quad$ $S_3 = {}_{j=1}^{2}\tau_3 \{ \underline{{}_{i=1}^{4}\tau_2 \mathbf{[x_{ei,j}, 90^o, a_5)]}}, a_3\}$

$S_4 = {}_{i=1}^{4}\tau_2 \{x_{ei}, (90^o, a_5)\}$

S_4 S_1 and S_4 S_3

Recursion: A schema is a recursive schema when it embeds within itself. If a shape pattern schema shares the same patterns with every levels of a shape pattern, then the first shape pattern is embedded into the second shape pattern schema recursively. For example, a shape pattern schema can be generalised from two shape patterns N_7 and N_8 in Figure 37, and the shape pattern schema S_5 is a rotation of rotation pattern, represented as $S_5 = {}_{j=1}^{4}\tau_2 \{ {}_{i=1}^{4}\tau_2 [x_{ei,j}, (a_2, a_5)], (a_2, a_5)\}$. The rotation pattern schema S_4 appears recursively in two levels of the shape pattern schema S_5. The shape pattern schema S_4 is embedded in the shape pattern schema S_5.



$N_7 = {}_{i=1}^{4}\tau_2\{ {}_{i=1}^{4}\tau_2 [e_{i,j}, (a_2, a_5)], (a_2, a_5)\}$ $\quad$ $N_8 = {}_{j=1}^{4}\tau_2 \{ {}_{i=1}^{4}\tau_2 [e_{i,j}, (a_2, a_5)], (a_2, a_5)\}$

$S_5 = {}_{j=1}^{4}\tau_2 \{ \underline{{}_{i=1}^{4}\tau_2 [x_{ei,j}, (a_2, a_5)]}, (a_2, a_5)\}$ $\quad$ $S_6 = \underline{{}_{i=1}^{4}\tau_2 \{x_{ei,j}, (a_2, a_5)\}}$

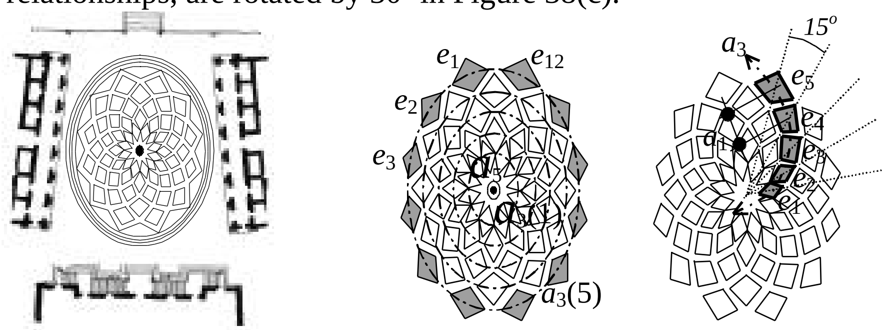
S_6 S_5

Figure 37. Shape patterns and their recursive schemas.

6.3 Multiple representations

Schemas are constructed from repeated structural properties (or common structural properties) in a class of objects. A single object in a class can be represented in terms of different attributes, in addition, many different representations are possible by perceivers using different composition methods from elements.

Primitive shapes are identified from the shape input data as well as emerged shapes. Primitive shapes are similar shapes to shapes stored as predefined knowledge, or peculiar shapes that are easily recognised. Properties of primitives are compared and similar shapes are grouped together and their spatial relationships are identified using inductive learning processes (Michalski, 1983). Various shape pattern descriptions can be constructed from single shape objects in terms of different intentions and predefined shape knowledge. For example, two different sets of quadrilaterals are recognised in Figure 38. Five sets of quadrilaterals can be identified, and each shape group is composed of 12 quadrilaterals with a rotation and a translation relationship on an oval in Figure 38(b). These five group shapes have the same relationships with each other, a rotation and a scale relationship. Furthermore, group shapes, that are composed of five quadrilaterals with 15^o rotation, translation on an axis a_3 and scale relationships, are rotated by 30^o in Figure 38(c).



(a) (b) (c)

$$(b) = {}_{j=1}^{5}\tau_2 \{\tau_4 \{ {}_{i=1}^{12}\tau_2 [\tau_1 (e_{i,j}, (a_1, a_3(i))), (30^o, a_5)], a_4\}, (15^o, a_5)\}$$

$$(c) = {}_{j=1}^{12}\tau_2 \{ {}_{i=1}^{5}\tau_2\{\tau_4 [\tau_1 (e_{i,j}, (a_1, a_3)), a_4], (15^o, a_5)\}, (30^o, a_5)\}$$

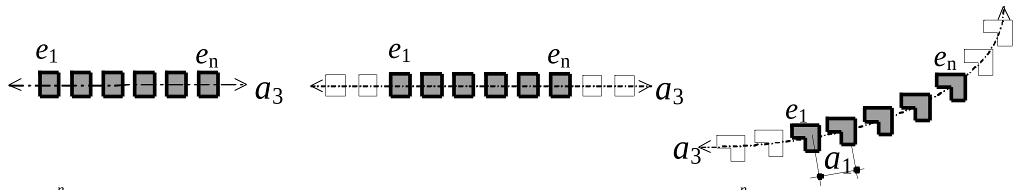
Figure 38. Multiple representations from a single shape: (a) Campidoglio by Michelangelo, (b) and (c) two different representations for the drawing in (a).

Repeated shape schemas or shape descriptions extracted from multiple shape representations can be used for schema construction from a class of shape objects. In Figure 38, two representations from a single object share the rotation relationship of twelve elements, and the rotation and scale relationship of five elements. They are shape schemas constructed from multiple representations. This multiple representation can provide various interpretations from a single designed object.

6.4 Formative idea representation using shape pattern representation

A shape pattern is a repeated arrangement of similar shapes or relationships. It is arranged in a regular form. So far properties and representations of obvious shape patterns have been investigated. However, shape patterns are artistically distorted or manipulated in many architectural designs. Some patterns are expected to be continued. Irregular elements are inserted so that the shape pattern knowledge in architectural drawings becomes obscured. Also groups of patterns may be mixed and permutated. Those deformations of shape patterns should be identified and operationally symbolised to represent them. Some deformed formative shape patterns may have meanings, such as continuation, grid or array, alienation and bending. Their shape pattern representations and examples are given below.

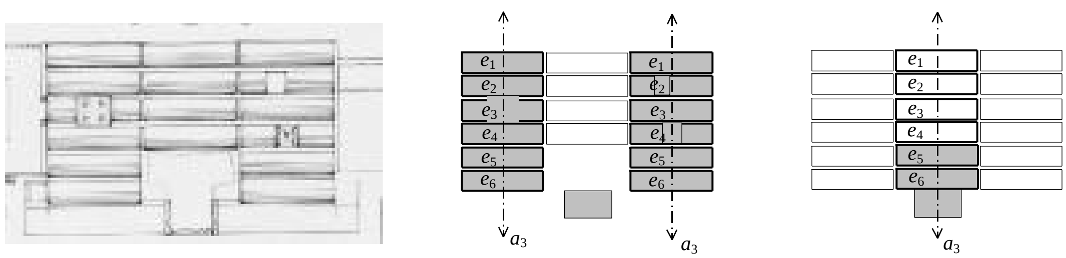
Continuation (τ_5): Shapes repeated in a certain pattern can be expected to be continued by shape pattern emergence. A translated shape pattern on an axis can have more shapes expected before and after the shape group. This is represented as $\tau_5\ {}_{i=1}^{n}\tau_1\{e_i, (a_1, a_3)\}$ in Figure 39.



${}_{i=1}^{n}\tau_1\{e_i, (a_1, a_3)\}$ **Continuation** = $\tau_5\ {}_{i=1}^{n}\tau_1\{e_i, (a_1, a_3)\}$

Figure 39. Continuation representation.

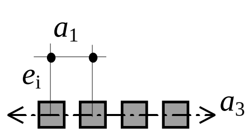
For example, the Kimbell Museum in Figure 40(a), designed by Louis I. Kahn, is composed of rectangles. Six rectangles are arranged in the form of a translation pattern in the right and left wings {${}_{i=1}^{6}\tau_1[e_i, (a_1, a_3)]$} in Figure 40(b). At the centre in Figure 40(c), only four rectangles are translated {${}_{i=1}^{4}\tau_1[e_i, (a_1, a_3)]$}, and two shapes ($e_5$ and e_6) can be emerged by continuation. It is represented as follows: **Continuation** = $\tau_5\ {}_{i=1}^{4}\tau_1\{e_i, (a_1, a_3)\}$



(a) (b) (c)

Figure 40. (a) Kimbell Museum in Fort Worth (Louis I. Kahn), (b) translation pattern of four rectangles, and (c) two rectangles emerged by continuation.

Grid or Array: In a grid or array space arrangement, elements in rows or columns are grouped together and a translation pattern is specified [${}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$]. All patterns are congruent such that they are grouped together again and identify a high-level pattern that is a translation of translation pattern, Figure 41.



$S_1 = {}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$

$S_2 = {}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$

. . .

$S_j = {}_{i=1}^{m}\tau_1\{e_i, (a_1, a_3)\}$

$e_{i,j}$ **Grid** = ${}_{j=1}^{n}\tau_1\{S_j, (a_1, a_3)\}$ or ${}_{j=1}^{n}\tau_1\{{}_{i=1}^{m}\tau_1[e_{i,j}, (a_1, a_3)], (a_1, a_3)\}$

Figure 41. Grid or array pattern representation.

For example, the Kimbell Museum in Figure 40(c) has a distinctive formative idea that is a grid pattern. Three rectangles ($e_{1,j}$, $e_{2,j}$, $e_{3,j}$,) in a row specify a translation pattern (${}_{i=1}^{3}\tau_1[e_{i,j}$,

(a_1, a_3)]). Then similar six translation patterns are translated again and specify a grid pattern ($\sum_{j=1}^{6}\tau_1\{\sum_{i=1}^{3}\tau_1[e_{i,j}, (a_1, a_3)], (a_1, a_3)\}$), Figure 42.

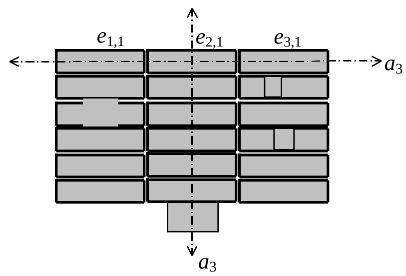


Figure 42. Grid pattern in the Kimbell Museum.

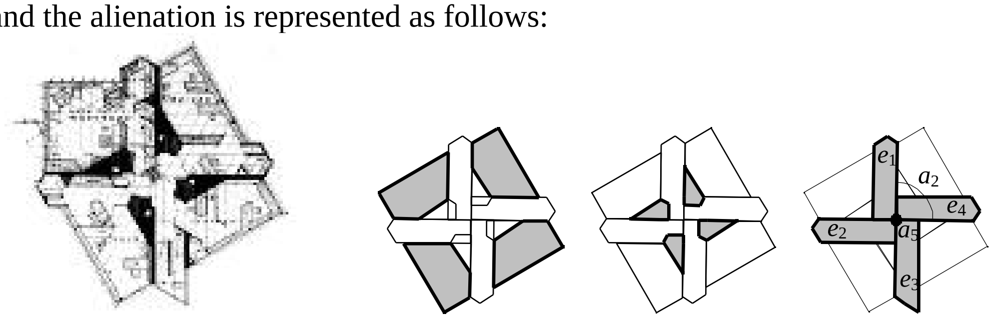
Alienation (τ_6): A set of the same shapes in a pattern may have a small number of different shapes introduced to it. Describing such a shape pattern may have difficulties due to this shape difference. They can be resolved through the introduction of the concept of alienation denoted by τ_6. For example, squares are arranged in a grid containing one shape which is not a square, that is an alienated shape. To represent this shape group, the circle should be regarded as a continued square and the location of the alienated circle need to be specified. This is represented as a grid pattern with an alienated circle ($e_{p,q}$), Figure 43.

$$\text{Grid} = \sum_{j=1}^{n}\tau_1\{\sum_{i=1}^{m}\tau_1[e_{i,j}, (a_1, a_3)], (a_1, a_3)\}$$

$$\textbf{Alienation} = \tau_6\{\text{Grid}, e_{p,q}\}$$

Figure 43. Grid with alienation pattern representation.

For example, the drawing in Figure 44(a) is the Price Tower designed by Frank Lloyd Wright. It is composed of three groups of shapes, Figures 44(b), (c) and (d). These three group shapes are arranged in a pattern that is a 90° rotation {$\sum_{i=1}^{4}\tau_3[e_i, (a_2, a_5)]$}. Only the shape e_3 in Figure 44(d) is different from other shapes in the pattern. It is alienated from this pattern and the alienation is represented as follows:

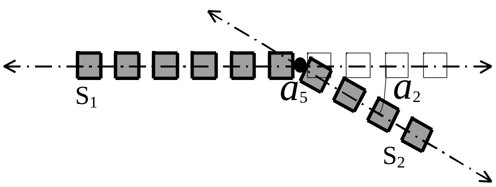


(a) (b) (c) (d)

$$\text{Pattern of (d)} = \tau_6\{\sum_{i=1}^{4}\tau_3[e_i, (a_2, a_5)], e_3\}$$

Figure 44. (a) The Price Tower (Frank Lloyd Wright), (b), (c) and (d) 90° rotation pattern with alienated shape.

Bending (τ_7): Two shape pattern descriptions have the same predicates and arguments, but they may have different numbers of elements and different axes, and their axes meet at a_5. A regarded as a continuous translation of squares by the continuation function. A shape pattern S_2 is also a translation of four squares along the axis. If the shape pattern S_2 rotated by the angle a_2 is embedded in the continued shape pattern S_1, these two shape groups can be considered as a single shape pattern with bending at the point a_5 with the angle a_2 in Figure 45.



$S_1 = \sum_{i=1}^{6} \tau_1\{e_i, (a_1, a_3)\} \rightarrow S'_1 = \tau_6\{\sum_{i=1}^{6} \tau_1[e_i, (a_1, a_3)]\}$

$S_2 = \sum_{j=1}^{4} \tau_1\{e_j, (a_1, a_3)\} \rightarrow S'_2 = \tau_2\{S_2, (a_2, a_5)\}$

If Embed(S'_2, S'_1) then

Bending = $\tau_7\{(S_1, S_2), (a_2, a_5)\}$

Figure 45. Bending representation.

For example, the Villa Kokkonen designed by Alvar Aalto in Figure 46 can be represented with a set of rectangles. Three rectangles (e_1, e_2, e_3) are translated along the axis ($a_{3,1}$) and another three rectangles (e_4, e_5, e_6) are arranged along the axis ($a_{3,2}$). The relationships of these two shape groups are similar and the one (S_1) is embedded in the other (S_2). Thus two shape patterns can be regarded as one shape pattern with bending on the point a_5 with the rotation angle a_2. Thus two shape patterns can be regarded as one shape pattern with bending on the point a_5 with rotation angle a_2.

(a) (b)

$S_1 = \sum_{i=1}^{3} \tau_1\{e_i, (a_1, a_3)\}$ $S_2 = \sum_{j=4}^{6} \tau_1\{e_j, (a_1, a_3)\}$

Villa Kokkonen = $\tau_7\{(S_1, S_2), (a_2, a_5)\}$

Figure 46. (a) Villa Kokkonen (Alvar Aalto), and (b) bending representation.

7 Applications of shape pattern representation in design

Using a schema approach, formative shape patterns in architectural drawings have been recognised and represented. These shape pattern representations are fundamental in many areas in design computation, such as style learning, analogical reasoning with shapes, and measurement of shape complexity.

In style learning, the computer learns a style representation from a set of shape descriptions belonging to a class using inductive generalisation processes. At first, commonalities are identified from shape descriptions that are composed of shape elements and their relationships in terms of physical shape elements and spatial relationships. Then these commonalities are generalised using inductive generalisation rules, such as dropping condition rules, turning constants into variables rules and climbing generalisation tree rules (Michalski, 1983). The generalised commonalities are learned style representations that characterise the class. Sets of learned style representations can be used in classification and categorisation. There are two kinds of style in terms of commonalities, prototype style and family style. Prototype style is classified by common features that cover all the objects in a class. However, this is not enough to explain the style. Wittgenstein (1960) suggested that family resemblance as well as conjunction is linked to determine a category. Family style is specified by disjunctions of class members. Interpretation of prototype style and family style can produce various design results. A base shape can be constructed from the combination of prototype schemas and shapes for shape generation. Then, the addition of family style and modification give the base shape more details and produces results that have properties of the style.

In analogical reasoning, the shape schema is a fundamental representation for structure information. According to Vosniadou and Ortony (1989), the mechanism of analogical reasoning is the identification and transfer of structural information from a known system (the source) to a new and relatively unknown system (the target). Learned shape patterns stored in memory are retrieved if they have a similarity with the target design system in some defined way. Then structural knowledge (pattern) is mapped onto the target. Mapping processes can be either within domains or between domains. The applicability of this relational structure for the target should be evaluated. Analogical reasoning can support routine design, innovative design and creative design.

Understanding, interpretation and learning about shape patterns are fundamental procedures for design computing. Recognition of shape elements and their organisation supports interpretation and learning processes. Recognition processes may identify only a part of shape knowledge or superficial knowledge due to complexity of shapes. Complex shapes are made up of a large number of subshapes that interact in a non-simple way. Subshapes in a whole designed shape are not arranged by mere chance, but organised and ordered. Identification of those orders may cause a decrease of visual complexity. Complexity and simplicity of shapes may be measured in terms of normalising the length of shape pattern descriptions. But an increase in order does not necessarily means a decrease in complexity. Measurement of the complexity would be useful in the evaluation of some form of aesthetics.

Patterns as invariant knowledge help designers making decision in solving problems (Alexander, 1979) and can be adapted to new design situations with instantiations of low-level variables. Patterns are sometimes expressed as rules of thumb in design. A design started with many accumulated patterns is likely to produce faster and reliable solutions rather than starting from chaotic situations. Recognition and representation of patterns from previous designs are important for designing.

8 Conclusion

Based on the notion that the design is the application of previous design knowledge to a new design situation, design knowledge representation, particularly shape pattern representation, has been investigated to support design computation. Representation of design knowledge can help the designer's understanding and interpretation, and the computer's use of design concepts of previous design works. Interpretations from appropriate design knowledge representations could support innovative and creative design.

Shape pattern knowledge is an organised body of knowledge about spatial relationships between shapes. Schema representation has been employed to represent shape pattern knowledge. Emergent shape properties identified from emergent shape and relationships, multiple representations, and multiple levels, introduce new schemas and consequently new variables. Shape pattern knowledge is constructed in a hierarchical tree structure based on repetitions of similarities as well as congruency of shapes and relationships. It also has some properties conducive to generalisation, such as variability, embeddedness and multiple representations.

Shape patterns encapsulate formal design knowledge which can be interpreted from design results. Instantiation or application of shape pattern schemas as abstract design knowledge in various contexts can produce various design results. Emerged new schemas and new variables provide the possibility of creative design for the computer (Gero, 1992), as well as a new way of perceiving objects.

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